CHAPTER 7 – Introduction to Financial Derivatives

7.1 BID AND ASK PRICES

A **market maker** is an entity that facilitates the buying and selling of stocks, or other types of assets. A market maker is generally not interested in making a profit by speculating on the future prices of the assets that they deal will, but instead make money by charging small commissions to any individual using their services to conduct a trade, whether that individual is buying an asset or selling an asset. Some market makers will charge a fixed, or flat, commission for every trade, regardless of the size of the trade, whereas other market makers will set their commissions as a percentage of the value of the overall trade.

When an individual is interested in buying or selling an asset, they submit their request to a market maker along with a price at which they are willing to buy or sell the asset. The market maker than matches compatible buyers and sellers. When looking at the price of an asset on such a service, two prices will be listed: The bid price, and the ask price. These prices are explained below.

- The **bid price** of an asset is the highest price being offered by individuals interested in buying the asset. This is the price at which you could immediately sell the asset.
- The **ask price** of an asset is the lowest price being offered by individuals interested in selling the asset. This is the price at which you could immediately buy the asset.

Note that the listed ask price is always higher than the listed bid price. The difference between the two prices is called the **bid-ask spread.**

Example 7.1

A market maker has 3 offers from individuals wishing to sell a particular asset and 4 offers from individuals wishing to buy the asset. The prices associated with the offers are as follows:

- Sell offers: 30.95, 30.64, 31.24 Buy Offers: 29.97, 30.24, 30.13, 29.83

Determine the bid price, the ask price, and the bid-ask spread.

Example 7.2

Two stocks have bid and ask prices as follows:

- Stock A: Bid price of 60.75, Ask price of 61.50
- Stock B: Bid price of 82.70, Ask price of 83.10

An investor sells two shares of Stock A and buys one share of Stock B. The investor was charged a commission of 0.4% by the market maker. Calculate the investor's net costs.

Example 7.3

On January 1, the bid price of Stock XYZ is 214 and the ask prices is 214.30.
On August 1, the bid price of Stock XYZ is 192.60 and the ask price is 193.
An investor purchased 30 shares of Stock XYZ on January 1, and sold all 30 shares of August 1. The broker charged a flat commission of 15 for each trade. Calculate the investor's net losses, ignoring interest.

7.2 SHORT SALES

A **financial derivative** is an asset whose value is in some way linked to the value of some underlying asset. Derivatives can be bought and sold without either party needing to actually own the underlying asset. There are many different types of derivatives available, some of which increase in value if the underlying asset increases, and some that decrease in value if the underlying asset increases. This leads to the following two definitions.

- An investor is said to hold a **long position** in an asset if they either own the asset outright, or if they own a derivative whose value is positively related to that of the underlying asset. Individuals in a long position hope for the value of the asset to increase.
- An investor is said to hold a **short position** in an asset if they own a derivative whose value is negatively related to that of the underlying asset. Individuals in a short position hope for the value of the asset to decrease.

Short Sales

One of the most common types of derivatives is a short sale. As the name implies, a short sale represents a short position with respect to the underlying asset, which is typically a stock. One would enter a short sale on a stock if they expect the value of the stock to decline. The basic mechanics of a short sale are explained below.

1. The short seller borrows one share of the underlying stock. The lender of the stock is typically a broker. The short seller is required to return the share at some specified time T. This is called closing the short.
2. The short seller immediately sells the stock for the current stock price, S_0 .
3. At time T, the short seller is required to repurchase the stock at its current price S_T , and then return it to the original lender, thus closing the short.
4. If the stock price has decreased in the interim then $S_T < S_0$ and the short seller will receive a net profit of $S_0 - S_T$, ignoring interest. If the short-seller receives interest on the proceeds of the short sale at an effective annual rate of i , then their profit will be $S_0(1+i)^T - S_T$

Example 7.4

Josh enters a short sale on a stock whose current value is 120. He invests the proceeds from the sale into an account earning an annual effective rate of 3%. He closes the short 4 months later.

a) Determine Josh's net profit if the price of the stock is 105 when the short is closed.
b) Determine Josh's net profit if the price of the stock is 145 when the short is closed.

Risks of Short Selling

When you buy a stock, the most you can lose is the price you paid for the stock. However, there is theoretically no limit to how high the price of a stock could rise during the course of a short sale, and thus there is no limit to the potential losses incurred by a short seller.

Margin

Since there is no limit to the potential price of a stock when it come time to close a short sale, there is a substantial risk to the lender of the stock that the short seller might not be able to afford to repurchase the stock in order to close the short. To offset this risk, the lender often holds the proceeds from the sale of the stock until close, and might also ask for the seller to deposit some amount of collateral in case the price of the stock increases. This collateral is often called **margin** or a **haircut**. The seller generally earns interest on their posted margin, although short sellers do not generally earn interest on the proceeds of the short sale itself.

Example 7.5

Anthony sells a stock short for 8000. The proceeds of the sale are retained by the lender until close (and do not accrue interest). Anthony is required to post margin equal to 75% of the value of the sale. The lender pays interest at an annual effective rate of 4% on the margin account. Anthony closes the short 9 months later. At that time, the price of the stock is 7200. Determine Anthony's annual effective yield on the short sale.

Dividends

Assume that the underlying stock pays a dividend during the course of a short sale. At this point, neither the short seller or the lender of the stock actually owns the stock, and thus neither of the two will receive the dividend. In this scenario, the short seller is generally required to pay the dividend amount to the lender of the stock. Since this transaction typically occurs when the short is closed, it might be necessary for the dividend to be paid back with interest.

Example 7.6

Consider the scenario in Example 7.5. Assume now that the stock pays a dividend of 250 six months after the short sale was initiated. Anthony is required to pay the dividend back at close, along with interest on the dividend, calculated at 4% annual effective. Determine Anthony's annual effective yield.

Example 7.7

Fenton sells a stock short for 1125. The proceeds of the short sale are held by the lender until close, and do not collect interest. Fenton is required to post margin of 80% and is paid interest at an annual effective rate of 5% on his margin account. The short is to be closed exactly one year later. The day before the short is closed, the stock pays a dividend of 20. Fenton's annual effective return on the short sale is -12%. Determine the price of the stock at close.

7.3 FORWARD CONTRACTS

A **forward contract** is an agreement between two individuals in which one party agrees to buy an asset from the other party for a predetermined price on a predetermined date. The price of the asset is decided upon when the contract is entered into, but is not paid until the transaction actually takes place. Some terminology relating to forward contracts is provided below.

- The **expiration date** is the date on which the actual sale will take place.
- The **forward price** is the amount that will be paid for the asset on the expiration date.
- The party obligated to purchase the asset benefits if the value increases, and is thus in a long position with respect to the underlying asset. As such, we say that the buyer has entered into a **long forward**.
- The party obligated to sell the asset benefits if the value decreases, and is thus in a short position with respect to the underlying asset. As such, we say that the seller has entered into a **short forward**.
- A **spot price** is the price of the asset on any specific date (most importantly at expiration).
- The **payoff** to either party involved in a forward contract is the value of the contract to that party on the expiration date. If the forward price is F and the spot price at expiration is S_T, then the payoffs are:
  - **Long Forward Payoff:** $S_T - F$
  - **Short Forward Payoff:** $F - S_T$

Example 7.8

Jack enters into a long forward contract on a stock. If the price of the stock is S at expiration, then Jack's payoff would be 30. If the price of the stock at expiration is $1.2S$, then Jack's payoff would be 46. Determine the forward price.

Cash Settlement

It is often the case that the parties involved in a forward contract will opt for a cash settlement at expiration rather than actually transferring the asset. For instance, if the spot price of the asset at expiration is 120 and the forward price is 100, then the short party might simply pay the long party 20. This is generally done to cut down on transaction fees associated with transferring the asset. When using a cash settlement, the seller is not actually required to own the asset, and might be entering the contract purely for speculative purposes. If the long party actually wants to buy the asset and is not simply engaging in speculation, then they can combine the payoff with the forward price (that never changed hands) and simply buy the asset on the market.

Uses of Forward Contracts

As indicated above, forward contracts are often used for speculative purposes. If you expect the value of an asset to either increase or decrease, then you could enter into a long or short forward as appropriate. Forward contracts are also used as a tool to hedge against potential price increases. If, for instance, a manufacturer knows that they will need to purchase a certain amount of a resource one year from now and suspects that prices for the resource will increase over the next year, then they might use a forward contract to lock in a certain price.

Pricing Forwards on Non-Dividend-Paying Stocks

Two parties entering into forward contract are free to set any forward price that they wish. However, forward prices on stocks are readily bought and sold on the open market, and as a result their prices are set by the market. In fact, if the current risk-free annual effective rate of interest is i and the current value of the stock is S_0, then the price of a forward contract on the stock expiring in T years is equal to $F_{0,T} = S_0(1+i)^T$. When dealing with forward contracts, it is common to see the risk-free rate represented as a continuously compounded rate, r. In this case, the forward price is given by $F_{0,T} = S_0 e^{rT}$.

To understand why forward prices on stocks much be set as stated, consider the following example.

Example 7.9

The current price of a non-dividend stock is 100. The annual effective risk free rate is 6%.

a) Find the correct price of a one-year forward on the stock.

b) Assume that Nikki has an opportunity to enter into a one-year short forward on the stock with a forward price of 107. On the day she enters the forward, she borrows 100 which she uses to buy one share of the stock. She repays the 100 (with interest) on the expiration date for the forward. The price of the stock at expiration is S. Determine Nikki's net cash flows at time 0 and at time 1.

c) Assume that Melvin owns one share of the stock, and finds an opportunity to enter into a one-year long forward on the stock with a forward price of 105. On the day he enters the forward, Melvin sells his share of the stock and invests the proceeds at the current risk-free rate. He withdraws the money at the time of expiration for the forward. The price of the stock at expiration is S. Determine Melvin's net cash flows at time 0 and at time 1.

Arbitrage

The previous example describes two arbitrage scenarios. An **arbitrage** is a set of transactions that allows a party to generate a net positive cash flow at some point in time without any net negative cash flows, and without exposing themselves to any risk. A party who engages in arbitrage is called an **arbitrageur**. Arbitrageurs tend to force prices of forwards on the market to their correct values by capitalizing on any mispriced forwards.

Pricing Forwards on Dividend-Paying Stocks

When pricing a forward on a stock that pays dividends, one must subtract the future value of any dividends paid by the stock from the forward price of the stock. We consider examples involving stocks with discrete dividends, as well as stocks paying continuous dividends. Assume the current price of a stock is S_0 and the continuously compounded risk-free rate of interest is r. Consider a forward on the stock expiring at time T.

- If the stock pays discrete dividends, then $F_{0,T} = S_0 e^{rT} - \text{AV}(\text{Divs})$.
- If the stock pays continuous dividends at a rate of δ, then $F_{0,T} = S_0 e^{(r-\delta)T}$.

Example 7.10 The current price of a stock is 120. The stock is expected to pay a dividend of 8 six months from now, and another dividend of 8 one year from now. The continuously compounded risk-free rate of interest is 4%. Find the forward price of a forward contract expiring one year from now, immediately after the second dividend is paid.

Example 7.11 The current price of a stock is 160. The stock pays dividends continuously at a rate of 2%. The continuously compounded risk-free rate of interest is 5%. Find the forward price of a 9-month forward contract on the stock.

Example 7.12 The current price of a stock is 230. The stock is expected to pay a dividend of 7 in N months. Assuming a continuously compounded risk-free rate of 4%, the 15 month forward price of the stock is 234.70. Find N.

7.4 PREPAID FORWARD CONTRACTS

A prepaid forward contract is similar to a standard forward contract, except that the buyer pays the seller of the asset when the contract is entered into, as opposed to when the contract is fulfilled. As a result, the prepaid forward price of an asset, denoted by $F^P_{0,T}$, is equal to the present value of the forward price of the asset, $F_{0,T}$.

This allows us to consider four possible methods of buying a stock, based on when the payment is received and when the stock is delivered. The four methods are: buying the stock outright, borrowing to pay for the stock, using a prepaid forward, and using a forward contract. The details of these methods are given in the table below.

Method of Buying Stock	Time of Payment	Time Stock is Received	Notation for Pmt	Price (No Div.)	Price (Discrete Div.)	Price (Cont. Div.)
Outright purchase	0	0	--	S_0	S_0	S_0
Borrow to pay for stock	T	0	--	$S_0 e^{rT}$	$S_0 e^{rT}$	$S_0 e^{rT}$
Prepaid Forward contract	0	T	$F^P_{0,T}$	S_0	$S_0 - \text{PV}(\text{Divs})$	$S_0 e^{-\delta T}$
Forward contract	T	T	$F_{0,T}$	$S_0 e^{rT}$	$S_0 e^{rT} - \text{AV}(\text{Divs})$	$S_0 e^{(r-\delta)T}$

Example 7.13

The continuously compounded risk-free rate of interest is 6%. Stocks X, Y, and Z all currently sell for 50. Find the price of one-year forward contracts and one-year prepaid forward contracts for each of these three stocks if:

- Stock X pays no dividends.
- Stock Y is scheduled to pay a dividend of 1 in 6 months, and a dividend of 1 in one year (one day prior to expiration of the forward contract).
- Stock Z pays dividends continuously at a rate of 2%.

Example 7.14

A stock has a current price of S and is expected to pay a dividend of 3.5 per share in 6 months. The one-year forward price for the stock is equal to S plus 4.78. The one-year prepaid forward price is 8.15 less than the one-year forward price. Determine the prepaid forward price of the stock.

Forward contracts themselves have inherit value, and can be bought and sold prior to their expiration date. The following example illustrates this concept.

Example 7.15

Assume that the continuously compounded risk-free rate is 4%. On January 1, Stock XYZ has a value of 100. On that day, Gene enters into a two-year long forward contract on XYZ. Six months later, the price of the stock is 92 and Gene enters into an 18 month long forward contract on XYZ. After another six months, the price of the stock is 112. At this point, Gene decides to liquidate his positions in XYZ. Determine the value of each of Gene's forward contracts on this day.

7.5 FUTURES CONTRACTS

A **futures contract** is a very standardized, highly regulated variation to the standard forward contract. Futures and forward are priced in similar ways, but have a few differences. The most important difference between the two instruments is the process of marking-to-market that is undergone by a future.

Marking -to-Market

It is possible for two forward contracts with the same underlying asset and with the same expiration date to have two different market values if they were initiated at different times. This phenomenon was illustrated in Example 7.15. Futures contracts are standardized so that any two futures contracts with the same underlying asset and with the same expiration date will always have the same market value. This is accomplished through a process called **marking-to-market**. Both parties involved in a futures contract must post collateral into an account called a **margin account**. These margin accounts earn interest and are periodically adjusted (usually daily) to reflect changes in the value of the underlying asset. This process guarantees that the market value of the future on any given day is exactly the same as a new future created on that day with the same expiration date.

Additional Characteristics of Futures

Several characteristics common to futures are described below.

- **Notional Value.** A futures contract for a specific asset will generally consist of a fixed number of units of the asset. This quantity is called the notional value of the index.
- **Initial Margin.** To minimize risk of default, both parties involved in a future make a margin deposit. The initial amount of margin required is generally a percentage of the total value of the contract.
- **Maintenance Margin.** Parties are often required to keep a minimum balance in their margin accounts. This minimum is typically a percentage of the initial margin, and is called the maintenance margin.
- **Margin Call.** If a margin account falls below the maintenance margin, the owner is required to make an additional deposit to make up the shortfall. This request for additional margin is called a margin call.

Example 7.16

Stock index XYZ is currently valued at 160. Lex enters into 10 long futures contracts on the index. The notional value for each contract is 200. The initial margin requirement is 10%. Lex's margin account earns interest at a continuously compounded rate of 4% and his position is marked-to-market weekly. The value of the index is 165 at the end of the first week, 175 at the end of the second week, and 170 at the end of the third week. Find the balance of Lex's margin account at the end of the third week.

Example 7.17

Stock index ABC is currently valued at 300. Zoey enters into 20 long futures contracts on the index. The notional value for each contract is 250. The initial margin requirement is 18%, and there is a maintenance margin of 80%. Zoey's margin account earns interest at a continuously compounded rate of 6% and her position is marked-to-market weekly. The value of the index after the first week is X. Find the largest value of X that would result in Zoey receiving a margin call at the end of the first week