CHAPTER 8 – Introduction to Options

8.1 CALL OPTIONS

A **call option** is a type of derivative contract in which the owner of the option has the right, but not the obligation, to purchase the underlying asset for a preset price from the party who sold the option. The details of a call option are explained below.

- The purchaser or holder of the option has the right to decide whether or not to purchase the option at a predetermined time, called the **expiration date**, for a preset price, called the **strike price**.
- If the holder of the the option does decide to purchase the asset when the option expires, the we say that the option has been **exercised**, or that the holder has exercised his right to purchase the asset.
- The individual who sold the option is called the **writer** of the option. The writer of the option is obligated to sell the asset if the purchaser chooses to exercise.
- When buying an option, the purchaser must pay some amount of money to the writer of the option. That amount of money is called the **option premium**. Writers sell options to collect the premium.
- We denote the premium of a call with a strike price of K and with T years until expiration by $\text{Call}(K, T)$.

American vs. European Options

There are two different styles of options in common usage; European and American options. **Europeans options** are only able to be exercised on the date of expiration for the option. **American options**, on the other hand, are able to be exercised on any date up until the expiration date. We will work almost exclusively with European options in this course.

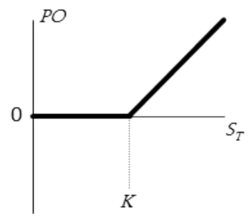
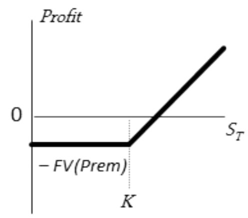
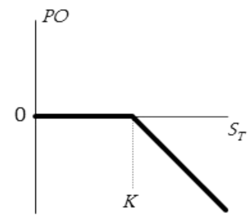
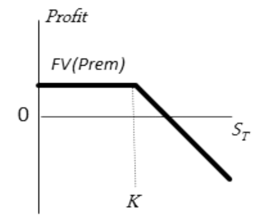
Position with Respect to the Underlying Asset

- **Long (Purchased) Call**. The purchaser of the option hopes that the asset will rise above the strike price so that he or she can purchase the asset at a reduced price. Thus, the purchaser of a call is long with respect to the underlying asset.
- **Short (Written) Call.** The writer of the option hopes that the asset will decrease, reducing the likelihood that the option will be exercised. Thus, the writer of a call is short with respect to the underlying asset.

Call Option Payoff and Profit

The **payoff** of an option for a certain party is the net gain or loss for that party. The **profit** at expiration for an option is the payoff up or down by the future value of the premium, depending on whether the party in question paid or received the premium. In the formulas for payoff and profit provided below, S_T denotes the spot price of the asset at expiration, while K denotes the strike price of the option.

- **Long Call Option:** $PO = \max[0, S_T - K]$, $\text{Profit} = \max[0, S_T - K] - FV(\text{Prem})$
- **Short Call Option:** $PO = -\max[0, S_T - K]$, $\text{Profit} = FV(\text{Prem}) - \max[0, S_T - K]$

<table>
<tr><th colspan="2">Long (Purchased) Call</th><th colspan="2">Short (Written) Call</th></tr>
<tr><th>Payoff</th><th>Profit</th><th>Payoff</th><th>Profit</th></tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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Example 8.1

The current price of one share of GlobalCorp stock is currently 115. Frank purchases a 9-month call on the stock with a strike price of 120. The premium for the call was 11.25. The currently continuously compounded risk-free rate is 4%. Find Frank's payoff and profit at expiration if:

a) The price of the stock at expiration is 150.
b) The price of the stock at expiration is 130.
c) The price of the stock at expiration is 110.

Example 8.2

Crystal purchases two one-year European calls on an asset. One of the calls has a strike price of 80 and a premium of 5.74, and the other call has a strike price of 90 and a premium of 3.29. Assuming a continuously compounded risk-free rate of 3%, Crystal's profit at expiration is equal to 6.695. Find the price of the asset at expiration.

Example 8.3

Doug writes a one-year European call option with a strike price of K and a premium of 15.80. The annual effective risk-free rate of interest is 5%. Doug breaks even on the investment if the spot price at expiration is 178.59. Find K.

Example 8.4

The current price of a stock is $62. Jason makes the following transactions:

- Purchase one 55-strike European call option with a premium of $13.41.
- Write two 60-strike European call options with a premium of $10.46.
- Purchase three 65-strike European call options with a premium of $8.03.
- Write three 70-strike European call options with a premium of $6.06.
- Purchase one 75-strike European call option with a premium of $4.52.

All options above have the same underlying stock and have 1 year until expiration.
The continuously compounded risk-free interest rate is 7%.

Calculate the maximum profit that Jason can obtain from this strategy.

Example 8.5

The spot price of a certain stock is currently \$80.
Grant purchases a one-year 85-strike European call on the stock for a premium of \$8.83.
Heidi writes a one-year 105-strike European call on the same stock for a premium of \$3.37.
The risk-free interest rate is 4%, compounded continuously.
At a spot price of S at expiration, Grant's profit is equal to Heidi's profit. Find S.

Example 8.6

The spot price of a certain stock is currently \$95.
Lori purchases a one-year 100-strike European call on the stock for a premium of \$11.69.
Chad purchases a one-year 120-strike European call on the same stock for a premium of \$5.39.
The risk-free interest rate is 6%, compounded continuously.
At a spot price of S at expiration, Lori's profit is equal to Chads's profit. Find S.

8.2 PUT OPTIONS

A **put option** is similar to a call, except that a put grants the owner of the option the right to sell the option for the strike price at expiration. The terminology relating to put options is directly analogous to that of call options.

- The purchaser or holder of the option has the right to decide whether or not to sell the option at a predetermined time, called the **expiration date**, for a preset price, called the **strike price**.
- If the holder of the the option does decide to sell the asset when the option expires, the we say that the option has been **exercised**, or that the holder has exercised his right to sell the asset.
- The individual who sold the option is called the **writer** of the option. The writer of the option is obligated to buy the asset if the purchaser chooses to exercise.
- When buying an option, the purchaser must pay some amount of money to the writer of the option. That amount of money is called the **option premium**. Writers sell options to collect the premium.
- We denote the premium of a put with a strike price of K and with T years until expiration by $\text{Put}(K, T)$.

Position with Respect to the Underlying Asset

- **Long (Purchased) Put**. The purchaser of a put option benefits from option if the price of the asset drops below the strike price. Thus, the purchaser of a put is short with respect to the underlying asset.
- **Short (Written) Put.** The writer of a put option hopes that the asset will increase, reducing the likelihood that the option will be exercised. Thus, the writer of a put is long with respect to the underlying asset.

Put Option Payoff and Profit

The **payoff** of an option for a certain party is the net gain or loss for that party. The **profit** at expiration for an option is the payoff up or down by the future value of the premium, depending on whether the party in question paid or received the premium. In the formulas for payoff and profit provided below, S_T denotes the spot price of the asset at expiration, while K denotes the strike price of the option.

- **Long Put Option:** $PO = \max[0\ ,\ K - S_T]$, $\quad$ $\text{Profit} = \max[0\ ,\ K - S_T] - FV(\text{Prem})$
- **Short Put Option:** $PO = -\max[0\ ,\ K - S_T]$, $\quad$ $\text{Profit} = FV(\text{Prem}) - \max[0\ ,\ K - S_T]$

<table>
<tr><th colspan="2">Long (Purchased) Put</th><th colspan="2">Short (Written) Put</th></tr>
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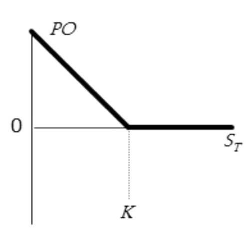
<td>
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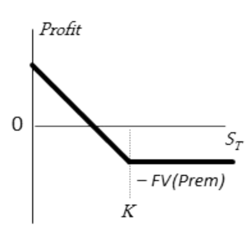
<td>
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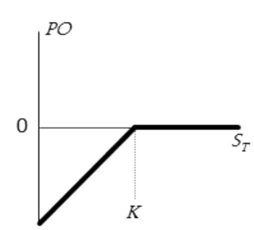
<td>
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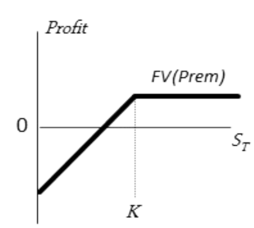
<td>
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Example 8.7

Lillian buys a one-year, 120-strike European put with a premium of \$10.86. The risk free rate of interest is 9.5% effective per annum. At a spot rate of S at expiration, Lillian's profit is 0. Determine S.

Example 8.8

Jonah buys a 6-month 110-strike European put with a premium of \$7.66. He also writes a 6-month 120-strike European put with a premium of \$13.20 on the same underlying asset. The risk-free rate of interest is 6% effective per annum. The spot price at expiration is \$112. Marge's total profit on the two options is X. Find X.

Example 8.9

The spot price of a certain stock is currently \$60.
Tim writes a one-year 50-strike European put on the stock for a premium of \$1.94.
Lars purchases a one-year 70-strike European put on the same stock for a premium of \$11.06.
The risk-free interest rate is 5.5%, compounded continuously.
At a spot price of S at expiration, Tim's profit is equal to Lars's profit.
Find S.

Example 8.10

The spot price of a certain stock is currently \$95.
Anna writes a one-year 90-strike European put on the stock for a premium of \$6.89.
Joan writes a one-year 105-strike European call on the same stock for a premium of \$9.15.
The risk-free interest rate is 4.5%, compounded continuously.
At a spot price of S at expiration, Anna's profit is equal to Joan's profit.
Find S.

8.3 PUT-CALL PARITY

The process of calculating the correct premium for a put or call option is complicated, and is a task that we are not yet ready to fully undertake. However, by comparing the present values of cash flows generated by certain types of derivatives, one can determine a relationship between European put and European call premiums. This relationship is called put-call parity. Three version of this relationship are stated below.

- **General Put-Call Parity:** $\text{Call}(K,T) - \text{Put}(K,T) = PV\left(F_{0,T}\right) - PV(K)$
  $\text{Call}(K,T) - \text{Put}(K,T) = F_{0,T}^{P} - PV(K)$
- **Put-Call Parity for Non-Dividend Stock:** $\text{Call}(K,T) - \text{Put}(K,T) = S_0 - PV(K)$

Note that the put-call parity relationship only holds for European options.

Derivation of Put-Call Parity

Consider two derivative portfolios:

- Portfolio A contains one long call and one short put on an asset. Both options have a strike price of *K*, and both expire at time *T*. Convince yourself that regardless of the spot price of the asset at expiration, the owner of this portfolio will end up purchasing the asset for *K* at time *T*.
- Portfolio B consists only of a prepaid long forward contract expiring at time *T*. The underlying asset for the forward is the same as for the options in Portfolio A.

Both portfolios will result in the owner of the portfolio receiving the asset at time *T*. In order to prevent arbitrage, the costs associated with the two portfolios should have the same present value.

- At time 0, the owner of Portfolio A will pay $Call(K,T)$ and will receive $Put(K,T)$. The owner of Portfolio A will also pay *K* for the asset at time *T*. Thus, the present value of the total cost to the owner of Portfolio A is $Call(K,T) - Put(K,T) + PV(K)$.
- The only cash flow for the owner of Portfolio B is a payment of $F_{0,T}^{P}$ at time 0.
- Setting the present values equal gives us $Call(K,T) - Put(K,T) + PV(K) = F_{0,T}^{P}$, which can be rewritten as $\text{Call}(K,T) - \text{Put}(K,T) = F_{0,T}^{P} - PV(K)$.

Example 8.11

The current spot price for a non-dividend-paying stock is $50. The premium for a 12-month European put with an exercise price of $55 on that stock is $6.50. The effective annual interest rate is 8%. Find the price of a 12-month European call option with a strike price of $55 on the same stock.

Example 8.12

The current forward price for a one-year forward on a certain stock is $287.92. The premium for a one-year 275-strike European call on the stock is $38.60, and the premium for a one-year 275-strike European put on the stock is $26.26. Determine the risk-free annual effective rate of interest.

Example 8.13 The forward price for delivery of one share of XYZ stock in one year is 137.35. The stock does not pay dividends. The continuously compounded risk-free rate of interest is 5.5%. A K-strike one-year European call option on one share of XYZ stock costs 24.17. A K-strike one-year European put option on one share of XYZ stock costs 7.75. Find K.

Hedging Strategies

Occasionally an investor with either a long or short position with respect to a certain asset might wish to enter into an option contract with the opposite position in the underlying asset as a means of providing insurance for the investment. Such a strategy is referred to as **hedging**. Four hedging strategies are described in the table below.

Strategy	Construction	Purpose	Equivalent Strategy (In Terms of Profit)
Protective Put	Long Asset + Long Put	The put places a floor on the amount that the owner is able to sell the asset for.	Long Call
Covered Call	Long Asset + Short Call	The call puts a cap on how much the owner can sell the asset for, but generates a premium that can offset potential losses.	Short Put
Covered Put	Short Asset + Short Put	The put places a floor on the what the short-seller will pay to close, but also generates a premium to offset losses if the price increases.	Short Call
Protective Call	Short Asset + Long Call	The call places a cap on the amount that the short seller is required to pay at close.	Long Put

Example 8.14 Carmen buys a share of stock for $50 and buys a 3-month 50-strike European put at the same time. The premium for a 3-month 50-strike European call is $3.29. The risk-free interest rate is 5% per annum compounded quarterly. Carmen has a profit of 0 at expiration. Find the spot price of the stock at expiration.

Example 8.15 Omar buys a stock for $70 and writes a 70-strike one-year European call on the same stock. The premium for a 70-strike one-year put is $6.34. The risk-free annual effective rate of interest is 5.8%. Find Omar's profit if the spot price at expiration is $77.

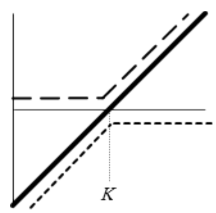
Example 8.16 Geoff sells a stock short for $55 and writes a 3-month European 55-strike put at the same time. The premium for a 3-month 55-strike call is $3.72. The risk-free rate of interest is 6.5% compounded quarterly. Geoff's overall profit is $1. Find the spot price of the asset at expiration.

Example 8.17 Myra sells a stock short for $50 and purchases a one-year European 50-strike call at the same time. The premium for a one-year 50-strike put is $2.62. The risk-free interest rate is 6% effective per annum. The spot price at expiration is $44. Determine Myra's profit.

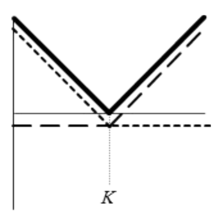
8.4 CONSTRUCTING SPREADS

In this section we will see how to combine puts and calls to construct a variety of financial instruments called spreads. You should be familiar with how to construct these spreads and how to calculate their payoff and profit. You should also understand the strategies for which one might use any particular spread.

Synthetic Forward

	Payoff Graph	Construction	Strategy
Synthetic Forward		• K - Strike Long Call • K - Strike Short Put	• Can be used to create an arbitrage opportunity when there is a mispriced forward contract available.

Straddle

	Payoff Graph	Construction	Strategy
Long Straddle		• K - Strike Long Call • K - Strike Long Put	• This spread is likely to be profitable if the volatility of the underlying asset is high.

A straddle is said to be "at the money" if K is equal to the current price of the stock.

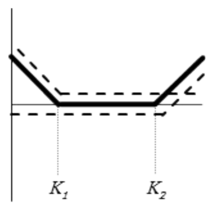
Example 8.18

The price of a certain stock is currently 100. A 100-strike 6-month European call on the stock has a premium of 10.35. A 100-strike 6-month European put on the stock has a premium of 6.50. The nominal risk-free rate of interest is 8%, convertible semiannually. Find the range of prices for which a 6-month at-the-money long straddle is profitable.

Example 8.19

Assuming a spot price at expiration of S, the writer of the straddle in Example 11.9 sees a profit of 8. Find the possible values of S.

Strangle

<table>
<tr><th></th><th>Payoff Graph</th><th>Construction</th><th>Strategy</th></tr>
<tr><td>Long Strangle</td><td>
</td><td>• K_1 - Strike Long Put
• K_2 - Strike Long Call</td><td>• Like a straddle, a strangle is profitable when there are large changes in the value of the asset.
• A strangle includes some degree of insurance against stable prices, in that it has a lower potential for loss than a straddle, but also a lower potential for gain.
• A strangle has lower financing costs than a straddle.</td></tr>
</table>

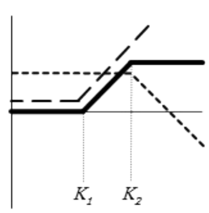
Example 8.20

The following premiums are for one-year European options for an underlying asset with a current spot price of 90.

Strike Price	Call	Put
80	16.99	3.09
90	11.10	6.71
100	6.87	11.99

The continuously compounded risk-free rate of interest is 5%. Determine the range of spot prices for which an at-the-money long straddle has a higher profit than a long 80-100 strangle.

Bull Spread

<table>
<tr><th></th><th>Payoff Graph</th><th>Construction</th><th>Strategy</th></tr>
<tr><td>Bull Spread</td><td>
</td><td>• K_1 - Strike Long Call
• K_2 - Strike Short Call</td><td>• A bull spread is long in the underlying asset.
• This spread places a cap on the potential losses, but also places a cap on the maximum potential profit.
• An investor might employ this strategy if they expect the asset price to increase only slightly.</td></tr>
</table>

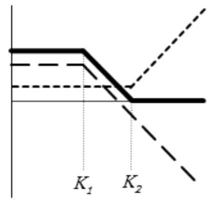
Example 8.21

Assume the conditions described in Example 11.20 still apply. Determine the range of prices at expiration for which an 80-100 bull spread has a positive profit.

Example 8.22

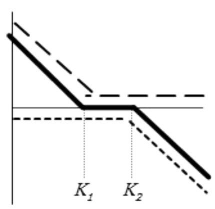
Assume the conditions described in Example 11.20 still apply. Determine the range of prices at expiration for which an 80-100 bull spread has a greater profit than a long 80-100 strangle

Bear Spread

	Payoff Graph	Construction	Strategy
Bear Spread		• K_1 - Strike Short Call • K_2 - Strike Long Call	• A bear spread is short in the underlying asset. • This spread places a cap on the potential losses, but also places a cap on the maximum potential profit. • An investor might employ this strategy if they expect the asset price to decrease only slightly.

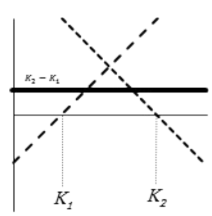
Example 8.23 Assume the conditions described in Example 11.20 still apply. Determine the range of prices at expiration for which an 80-100 bear spread has a greater profit than a written 80-strike call.

Collar

	Payoff Graph	Construction	Strategy
Collar		• K_1 - Strike Long Put • K_2 - Strike Short Call	• A collar is short with respect to the underlying asset. • Can be used to generate a small profit in premiums if the asset price does not change much. • A long or short collar can be combined with an opposing position in the underlying asset to create either a bull or bear spread.

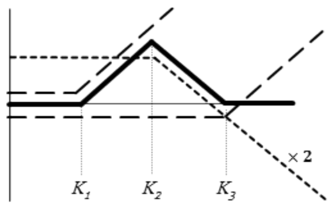
Example 8.24 Assume the conditions described in Example 11.20 still apply. Determine the range of prices at expiration for which an 80-100 written collar is profitable.

Box Spread

	Payoff Graph	Construction	Strategy
Box Spread		• K_1 - Strike Synthetic Fwd • K_2 - Strike Synthetic Fwd	• Generates a guaranteed return at the risk-free rate. • Can be used to take advantage of mispricings in option premiums.

Example 8.25 Assume the conditions described in Example 11.20 still apply. Find the cost of an 80-100 box spread.

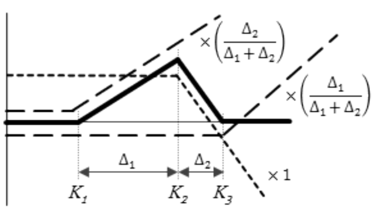
Butterfly Spread

	Payoff Graph	Construction	Strategy
Butterfly Spread		• K_1 - Strike Long Call • K_2 - Strike Short Call (× 2) • K_3 - Strike Long Call or • K_2 - Straddle (Short) • K_1 - K_3 - Strangle (Long)	• Similar to a written straddle, a butterfly spread is profitable when prices are stable. • A butterfly spread caps potential losses, providing insurance against high volatility.

Example 8.26

Assume the conditions described in Example 11.20 still apply. Determine the range of prices for which a written at-the-money straddle generates a higher profit than an 80-90-100 butterfly spread.

Asymmetric Butterfly Spread

	Payoff Graph	Construction	Strategy
Asy. Butterfly		• K_1 - Strike Long Call (× λ_1) • K_2 - Strike Short Call (× 1) • K_3 - Strike Long Call (× λ_2) $\lambda_1 = \Delta_2/(\Delta_1 + \Delta_2)$ $\lambda_2 = \Delta_1/(\Delta_1 + \Delta_2)$	• The strategies associated with an asymmetric butterfly spread are similar to that of a butterfly spread.

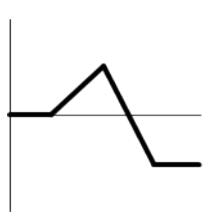
Example 8.27

The table to the right lists premiums for six-month European options for an underlying asset with a current spot price of 55.

An asymmetric butterfly spread is constructed with these options using the smallest whole number of options possible. Determine the range of prices for which this spread is profitable.

Strike Price	Call	Put
50	6.70	1.08
55	3.62	2.94
62	1.21	7.44

Ratio Spread

	Payoff Graph	Construction	Strategy
Ratio Spread		• Any nonstandard mix of calls and puts.	• ?