CHAPTER 9 – Swaps

9.1 COMMODITY SWAPS

A **commodity swap** is a contract that guarantees the delivery of an asset at a set of prescribed times for a certain preset price. It is similar to a forward contract, with the difference being that a forward contract has a single expiration date, whereas a swap is able to cover the several transactions all occurring at different times.

The goal of using a commodity swap to lock in the price of several different transactions could certainly be obtained with several forward contracts, each with a different expiration date and forward price. However, it is often desirable to have the payments involved in the swap contract to all be in the same amount. Such a swap is said to have **level payments**. The present value of the level swap payments must be equal to the present value of the forward prices.

Level Swap Prices

Let $F_{0,k}$ be the current price of a k-year forward on a certain commodity is denoted by . The level swap price L for an n-year swap is found by using the equation: $L\left(v + v^2 + ... + v^n\right) = F_{0,1}v + F_{0,2}v^2 + ... + F_{0,n}v^n$. If interest rates are described using spot rates rather than by an annual effective rate, then the previous equation becomes:

$$L\left(\frac{1}{1 + s_1} + \frac{1}{\left(1 + s_2\right)^2} + ... + \frac{1}{\left(1 + s_n\right)^n}\right) = \frac{F_{0,1}}{1 + s_1} + \frac{F_{0,2}}{\left(1 + s_2\right)^2} + ... + \frac{F_{0,n}}{\left(1 + s_n\right)^n} .$$

Example 9.1	The one, two, and three year forward prices for a certain commodity are currently 60, 64, and 70, respectively. The one, two, and three year sport rates are 5%, 5.4%, and 6%, respectively. Find the level swap price for a 3-year swap that guarantees delivery of one unit of the commodity at the end of each of the next three years.

Implicit Borrowing and Lending

Assume that two parties have entered into a swap contract with level payments. The level payments under the contract will not likely be equal to any of the forward prices for the commodity. As a result it is often assumed that a certain amount of lending and borrowing takes place between the two parties involved in a level swap.
- When the level payment is lower than what would have been expected with a forward, then we assume that the seller is loaning money to the buyer, and that the amount lent is equal to the difference between the forward and swap prices.
- When the level payment is higher than what would have been expected with a forward, then we assume that the buyer is loaning money to the seller, and that the amount lent is equal to the difference between the forward and swap prices.

Since forward prices typically increase as time until expiration increases, the level swap payment will be larger than the forward prices initially, but the swap payments will eventually be smaller than the forward payments. Thus, the buyer is the lending party at the early stages of the swap, but eventually becomes the borrower.

It turns out that the NPV of the amounts borrowed or lent by any party involved in the swap will be zero.

| Example 9.2 | Determine the implicit amounts borrowed or lent by the buyer in Example 9.1 at the end of each of the first three years. Determine the NPV of the cash flows. |

| Example 9.3 | The one and two-year forward prices of a commodity are 100 and 112, respectively. The two-year level swap price for the commodity is 105.80. The price of a one-year zero-coupon 1000-par bond is currently 943.40. Find the one-year forward rate. |

When working with swap, it is generally the case that the swap price will be level throughout the duration of the swap. That does not have to be the case, however. The swap prices could, in theory, be equal to any amounts that the two parties involved agree upon. However, it should always be true that the present value of the swap payments are equal to that of the forward prices.

| Example 9.4 | The one, two, and three year forward prices for a certain commodity are currently 80, 86, and 92, respectively. The one, two, and three year sport rates are 4.2%, 4.6%, and 5.2%, respectively. A 3-year swap guarantees the delivery of 100 units of the commodity at the end of each of the next three years. The terms of the swap stipulate that the swap price is to increase by 2% each year. Find the total amount paid by the buyer at the end of the first year. |

9.2 INTEREST RATE SWAPS

When studying amortization problems in the past, we almost always assumed that the interest rate for the loan was fixed. It is not uncommon, however, for loans to have "floating" rates that change over time. An **interest rate swap** is a contract that replaces a variable or floating rate with a single fixed rate.

Assume that a loan in the amount of 1 is to be repaid at the end of n years. Suppose that interest is to be repaid on the outstanding balance at the end of each year at the current forward rate for that year. Let R be the fixed swap rate for an n-year interest rate swap. Since we are assuming that the amount borrowed is 1, under the swap the loan will be repaid with n annual interest payments of R, followed by a final payment of 1. It follows that 1 is the present value of these payments. That is: $1 = \dfrac{R}{1 + s_1} + \dfrac{R}{\left(1 + s_2\right)^2} + ... + \dfrac{R}{\left(1 + s_n\right)^n} + \dfrac{1}{\left(1 + s_n\right)^n}$.

| Example 9.5 | A loan is to be repaid at the end of three year with interest payments due at the end of each year. The size of the interest payments are determined by the forward rates set by LIBOR (London Interbank Offered Rate). The current one, two, and three-year LIBOR spot rates are 5.6%, 5.9%, and 6.4%, respectively. Find the fixed rate for a three-year interest rate swap. |