Name: HW 1.4a Key

1. At an annual effective rate of 6%, the following two sets of payments have the same present value.
   i) A payment of 240 at time 2 and a payment of 280 at time 8.
   ii) A payment of 200 at time 3 and a payment of K at time 10.
   Find K.

[A] 396.41 B) 404.33 C) 412.26 D) 420.19 E) 428.12

$v = 1.06^{-1} = 0.94340$

$240v^2 + 280v^8 = 200v^3 + Kv^{10}$

$K = \boxed{396.41}$

2. You are given:
   (i) The sum of the present values of a payment X at the end of 15 years and a payment of Y at the end of 33 years is equal to the present value of a payment of $X + Y$ at the end of 19 years.
   (ii) $X + Y = 400$
   (iii) $i = 4\%$

   Calculate X. [1.a(i-v) #12]

[A] 285 B) 257 C) 271 D) 300 E) 314

$v = 1.04^{-1}$

$Xv^{15} + Yv^{33} = 400v^{19}$
$X + Y = 400$

$0.55526X + 0.27409Y = 189.857$
$0.27409X + 0.27409Y = 109.636$

$0.28117X = 80.221$

$X = \boxed{285.31}$

3. At an effective annual rate of i, $i > 0$, each of the following two sets of payments has present value K:
   (i) A payment of 400 immediately, and another payment of 400 at the end of one year.
   (ii) A payment of 441 at the end of 2 years, and another payment of 441 at the end of 3 years.
   Calculate K. [1.a(i-v) #06]

[A] 781 B) 750 C) 765 D) 797 E) 812

$K = 400 + 400v$
$K = 441v^2 + 441v^3$

$400 + 400v = 441v^2 + 441v^3$
$400(1+v) = 441v^2(1+v)$

$v^2 = \frac{400}{441}$

$v = 0.95238$

$K = 400 + 400v = \boxed{780.95}$

4. At an annual effective interest rate of i, $i > 0$, the following are all equal:
   (i) the present value of 3500 at the end of $2n$ years;
   (ii) the sum of the present values of 6000 at the end of year t and 26,000 at the end of year $2t$; and
   (iii) 2240 immediately.

   Calculate the present value of a payment of 17,500 at the end of year $t + n$ using the same annual effective interest rate. [1.a(i-v) #05]

   A) 2800 B) 2520 C) 2660 D) 2940 E) 3080

(i & iii) $2240 = 3500v^{2n}$

$v^{2n} = 0.64$

$v^{n} = 0.8$

(ii & iii) $2240 = 6000v^{t} + 26{,}000v^{2t}$

$26{,}000(v^{t})^{2} + 6000v^{t} - 2240 = 0$

$v^{t} = 0.2$

$17{,}500v^{t+n} = 17{,}500v^{t}v^{n} = \boxed{2800}$

5. John borrows 800 from Jane at an annual effective rate of interest i. He agrees to pay back 800 after 9 years and 1,233.18 after another 9 years.

   Instead of making the second payment as scheduled, John repays the outstanding balance 6 years after his first payment.

   What is the amount of John's second payment? [1.a.(i-v) #09]

   A) 1,007 B) 906 C) 931 D) 956 E) 981

$800 = 800v^{9} + 1233.18v^{18}$

$1233.18(v^{9})^{2} + 800v^{9} - 800 = 0$

$v^{9} = 0.54393$

$v = 0.93458 \rightarrow$ (and $i = 7\%$)

$800 = 800v^{9} + Xv^{15}$

$X = \boxed{1006.64}$