Name: HW 1.8 (a) Key

1. In Fund A, the accumulated value of 1 at any time $t > 0$ is $1+12t$.

   In Fund B, the accumulated value of 1 at any time $t > 0$ is $1+6t^2$.
   T is the time when the force of interest for Fund A is equal to the force of interest for Fund B.

   Calculate T. [1.b-f #20]

   [A] 0.333 B) 0.350 C) 0.367 D) 0.383 E) 0.400

$$\frac{12}{1+12t} = \frac{12t}{1+6t^2} \Rightarrow 1+6t^2 = t+12t^2 \Rightarrow 6t^2+t-1=0$$

$$\Rightarrow \boxed{t = 1/3}$$

2. You are given:

   (i) $A(t) = Kt^2 + Lt + M$ for $0 \le t \le 3$

   (ii) $A(0) = 200$

   (iii) $A(1) = 232$

   (iv) $A(3) = 368$

   Determine the force of interest at time $t = \frac{1}{2}$. [1.b-f #30]

   [A] 0.150 B) 0.135 C) 0.143 D) 0.158 E) 0.165

$200 = M$

$232 = K + L + 200$

$368 = 9K + 3L + 200$

$\rightarrow K + L = 32$, $3K + L = 56$ $\rightarrow 2K = 24$ $\rightarrow K = 12$, $L = 20$

$A(t) = 12t^2 + 20t + 200$

$A'(t) = 24t + 20$

$A(\frac{1}{2}) = 213$

$A'(\frac{1}{2}) = 32$

$\delta_{1/2} = \frac{32}{213} = \boxed{0.1502}$

3. Tawny makes a deposit into a bank account which credits interest at a nominal rate of 6% per annum, convertible semiannually.

At the same time, Fabio deposits 3000 into a different bank account, which is credited with simple interest.

At the end of 10 years, the forces of interest on the two accounts are equal, and Fabio's account has accumulated to *Z*.

Determine *Z*. [1.b-f #03]

A) 7338 B) 6971 C) 7705 D) 8072 E) 8439

Tawny: $i^{(2)} = 6\% \rightarrow j = 3\% \rightarrow i = 6.09\% \rightarrow \delta = 5.9118\%$

Fabio: $\delta_t = \frac{i}{1+it}$

$\frac{i}{1+10i} = 0.059118 \Rightarrow i = 0.059118 + 0.59118i \Rightarrow i = 14.46\%$

$3000(1+10(0.1446)) = \boxed{7338.12}$

4 At time 0, 1900 is deposited into Fund X and also into Fund Y. Fund X accumulates at a force of interest $\delta_t = 0.4(1+t)^{-2}$. Fund Y accumulates at an annual effective interest rate of *i*. At the end of 8 years, the accumulated value of Fund X equals the accumulated value of Fund Y.

Determine *i*. [1.b-f #06]

A) 4.5% B) 3.9% C) 4.1% D) 4.3% E) 4.8%

$\int_0^t 0.4(1+r)^{-2}\,dr = -0.4(1+r)^{-1}\Big|_0^t = 0.4\left[1-\frac{1}{1+t}\right] = \frac{0.4t}{1+t}$

$a(t) = e^{0.4t/(1+t)}$

$e^{3.2/9} = (1+i)^8 \rightarrow \boxed{i = 0.04545}$

5. At force of interest $\delta_t = \frac{0.5}{1+0.5t}$, $0 \le t \le 14$, the following payments have the same present value:

(i) *X* at the end of year 7 plus 2*X* at the end of year 13; and
(ii) *Y* at the end of year 19.

$a(t) = 1+0.5t$

Calculate *Y* / *X*. [1.b-f #08]

A) 5.13 B) 4.36 C) 4.62 D) 4.88 E) 5.39

$\frac{X}{a(7)} + \frac{2X}{a(13)} = \frac{Y}{a(19)} \rightarrow X\left(\frac{1}{4.5} + \frac{2}{7.5}\right) = \frac{Y}{10.5}$

$\rightarrow \frac{Y}{X} = \boxed{5.1333}$