# HW 1.8(b) Key

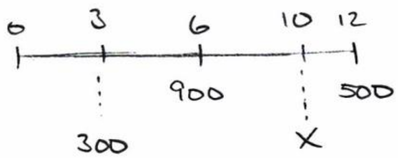
1. You are given $\delta_t = \dfrac{2}{1+t}$. $\rightarrow \int_0^t \frac{2}{1+r}\,dr = 2\ln|1+r|\Big|_0^t = 2\ln(1+t)$

A payment of 900 at the end of 6 years and 500 at the end of 12 years has the same present value as a payment of 300 at the end of 3 years and $X$ at the end of 10 years.

Calculate $X$. [1.b-f #22]

$$a(t) = e^{2\ln(1+t)} = (1+t)^2$$

$\boxed{\text{A)}}$ 312 B) 281 C) 296 D) 327 E) 343




$$\frac{900}{a(6)} + \frac{500}{a(12)} = \frac{300}{a(3)} + \frac{X}{a(10)}$$

$$\frac{900}{49} + \frac{500}{169} = \frac{300}{16} + \frac{X}{121}$$

$$X = \boxed{311.69}$$

2. A fund has a force of interest given by $\delta_t = \dfrac{0.09t}{1+0.045t^2}$. $\rightarrow a(t) = 1 + 0.045t^2$

A deposit is made into the account a time 6. Determine the number of years from the time of the deposit required for the value to double.

$\boxed{\text{A)}}$ 3.71 B) 3.47 C) 3.55 D) 3.63 E) 3.78


$$a(6 \rightarrow t) = 2 \rightarrow \frac{a(t)}{a(6)} = 2 \rightarrow 1 + 0.045t^2 = 2(2.62)$$

$$\rightarrow t = 9.7068$$

$$\Delta t = 9.7068 - 6 = \boxed{3.7068}$$

3. At time $t = 0$, Donald puts 4750 into a fund crediting interest at a nominal rate of $i$ compounded semiannually.

At time $t = 5$, Lewis puts 4750 into a different fund crediting interest at a force $\delta_t = \dfrac{1}{8+t}$ for all $t$.

At time $t = 14$, the amounts in each fund will be equal.

Calculate $i$. [1.g #06]

$\boxed{\text{A)}}$ 3.8% B) 3.6% C) 3.7% D) 3.9% E) 4.0%

$$a(t) = \tfrac{1}{8}(8+t)$$

$$a(14) = 2.75$$

$$a(5) = 1.625$$

$$\cancel{4750}\left(1+\frac{i}{2}\right)^{28} = \cancel{4750}\,a(5 \rightarrow 14)$$

$$\left(1+\frac{i}{2}\right)^{28} = \frac{a(14)}{a(5)} = \frac{2.75}{1.625}$$

$$i = \boxed{0.0379}$$

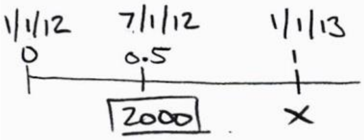
4. On July 1, 2012 a person invests 2,000 in a fund for which the force of interest at time $t$ is expressed by the formula $\delta_t = \dfrac{4+2t}{60}$ where $t$ is the number of years since January 1, 2012.

Determine the accumulated value of the investment on January 1, 2013. [1.g #16]

$\boxed{\text{A)}}$ 2094 B) 1989 C) 2198 D) 2303 E) 2408

$$\int_0^t \frac{4+2r}{60}\,dr = \frac{1}{60}\int_0^t (4+2r)\,dr = \frac{1}{60}\left[4r+r^2\right]_0^t = \frac{4t+t^2}{60}$$

$$a(t) = e^{(4t+t^2)/60}$$




$$2000\,\frac{a(1)}{a(0.5)} = 2000\left(\frac{1.0869}{1.0382}\right) = \boxed{2093.80}$$

5. Jeff puts 700 into a fund that pays an effective annual rate of discount of 12% for the first 3 years and of force of interest of rate $\delta_t = 2t/\left(t^2+5\right)$, $3 \le t \le 7$, for the next 4 years.

At the end of 7 years, the amount in Jeff's account is the same as what it would have been if he had put 700 into an account paying interest at the nominal rate of $i$ per annum compounded quarterly for 7 years.

Calculate $i$. [1.g #10]

$\boxed{\text{A)}}$ 0.255 B) 0.235 C) 0.276 D) 0.296 E) 0.317

$$\delta_t = \frac{2t}{t^2+5} \longrightarrow a(t) = \frac{1}{5}(t^2+5)$$

$$\left(1+\frac{i}{4}\right)^{28} = (0.88)^{-3}\,a(3\to 7)$$

$$\left(1+\frac{i}{4}\right)^{28} = (0.88)^{-3}\,\frac{54}{14}$$

$$i = \boxed{0.25546}$$