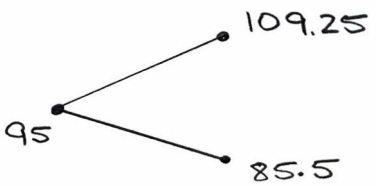
HW 10.3 Key

1. Prices for a stock are modeled with a 1-period binomial tree. You are given the following information:
   (1) The period is one year.
   (2) The stock's current price is 95
   (3) $u = 1.15$ and $d = 0.9$.
   (4) The continuously compounded risk-free rate is 3.5%.
   (5) The stock pays continuous dividends proportional to its price at a rate of 1.5%.
   Find the risk-neutral probability of an increase in the stock price. [20d_01]

   A) 48.08% B) 49.04% C) 50.00% D) 50.97% E) 51.93%



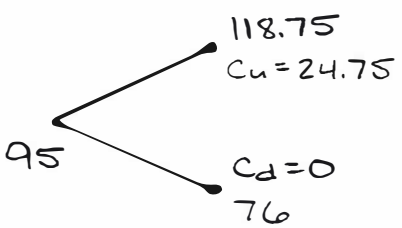
$$p^* S_u + (1-p^*) S_d = 95 e^{(r-\delta)t}$$

$$23.75 p^* + 85.5 = 95 e^{0.02}$$

$$p^* = \boxed{48.08\%}$$

2. Prices for a stock are modeled with a 1-period binomial tree. You are given the following information:
   (1) The period is one year.
   (2) The stock's current price is 95
   (3) $u = 1.25$ and $d = 0.8$.
   (4) The continuously compounded risk-free rate is 5%.
   (5) The stock pays continuous dividends proportional to its price at a rate of 2%.
   Use risk-neutral pricing to find the price of a one year call option with a strike price of 94. [20d_02]

   A) 12.06 B) 11.10 C) 11.58 D) 12.54 E) 13.02



$t = 1$ $r = 5\%$

$K = 94$ $\delta = 2\%$

$$118.75 p^* + 76 (1-p^*) = 95 e^{0.03}$$

$$p^* = 51.21\%$$

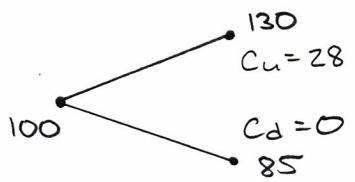
$$E[P0] = 24.75 p^* = 12.67$$

$$Call = 12.67 e^{-0.05}$$

$$= \boxed{12.06}$$

3. Prices for a stock are modeled with a 1-period binomial tree. You are given the following information:
(1) The period is 4 months.
(2) The stock's current price is 100
(3) $u = 1.3$ and $d = 0.85$.
(4) The continuously compounded risk-free rate is 4%.
(5) The stock does not pay dividends.
Use risk-neutral pricing to find the price of a 4-month call option with a strike price of 102. [20d_3]

[A] 10.03 B) 8.83 C) 9.23 D) 9.63 E) 10.43



$t = 1/3$ $r = 4\%$

$K = 102$ $\delta = 0\%$

$130p^* + 85(1-p^*) = 100e^{0.04/3}$

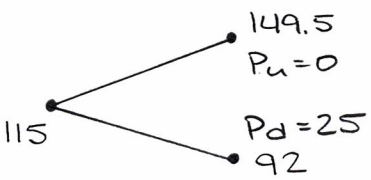
$p^* = 36.32\%$

$E[P0] = 28p^* = 10.17$

$Call = 11.87e^{-0.04/3} = \boxed{10.03}$

4. Prices for a stock are modeled with a 1-period binomial tree. You are given the following information:
(1) The period is one year.
(2) The stock's current price is 115
(3) $u = 1.3$ and $d = 0.8$.
(4) The continuously compounded risk-free rate is 4.5%.
(5) The stock does not pay dividends
Use risk-neutral pricing to find the price of a one year put option with a strike price of 117. [20d_04]

[A] 12.14 B) 10.20 C) 10.68 D) 11.17 E) 11.65



$t = 1$ $r = 4.5\%$

$K = 117$ $\delta = 0\%$

$149.5p^* + 92(1-p^*) = 115e^{0.045}$

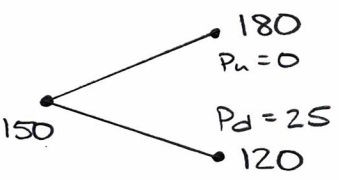
$p^* = 49.21\%$

$E[P0] = 25(1-p^*) = 12.70$

$Put = 12.70e^{-0.045} = \boxed{12.14}$

5. Prices for a stock are modeled with a 1-period binomial tree. You are given the following information:
(1) The period is 9 months.
(2) The stock's current price is 150
(3) $u = 1.2$ and $d = 0.8$.
(4) The continuously compounded risk-free rate is 4%.
(5) The stock pays continuous dividends proportional to its price at a rate of 2.5%.
Use risk-neutral pricing to find the price of a 9-month put option with a strike price of 145. [20d_05]

[A] 11.44 B) 10.07 C) 10.52 D) 10.98 E) 11.90



$t = 3/4$ $r = 4\%$

$K = 145$ $\delta = 2.5\%$

$180p^* + 120(1-p^*) = 150e^{0.015(0.75)}$

$p^* = 52.83\%$

$E[P0] = 25(1-p^*) = 11.79$

$Put = 11.79e^{-0.04(0.75)}$

$= \boxed{11.44}$