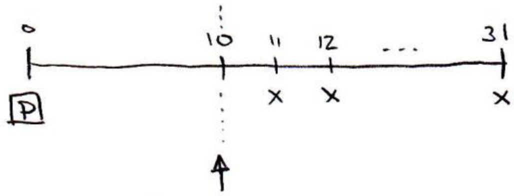
HW 2.2(b) Key

1. At time $t = 0$, Paul deposits P into a fund crediting interest at an effective annual interest rate of 7.5%. At the end of each year in years 11 through 31, Paul withdraws an amount sufficient to purchase an annuity-due of 150 per month for 9 years at a nominal interest rate of 8.1% compounded monthly. Immediately after the withdrawal at the end of year 31, the fund value is zero.

Calculate P. [3.a-c #04]

$\boxed{\text{A)}}$ 58,375 B) 57,208 C) 59,543 D) 60,710 E) 61,878



To find X:

$i^{(12)} = 8.1\% \rightarrow j = 0.00675$

$X = 150\,\ddot{a}_{\overline{108}|j} = 150(1+j)\,a_{\overline{108}|j}$

$= 11{,}553.5942$

$P(1.075)^{10} = X\,a_{\overline{21}|7.5\%}$

$P(1.075)^{10} = 120{,}313.1256$

$P = 58{,}375.20$

2. On January 1, an insurance company has 85,000 which is due to Linden as a life insurance death benefit. He chooses to receive the benefit annually over a period of 18 years, with the first payment immediately. The benefit he receives is based on an effective interest rate of 6.5% per annum.

The insurance company earns interest at an effective rate of 7% per annum. Every July 1, the company pays 140 in expenses and taxes to maintain the policy.

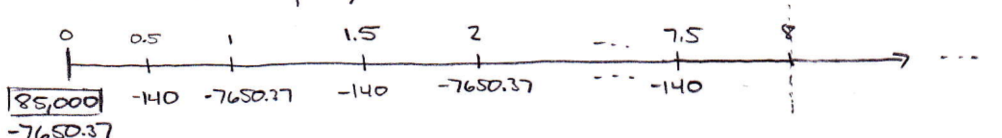
At the end of 8 years, the company has X remaining.

Calculate X. [3.a-c #11]

$\boxed{\text{A)}}$ 60,600 B) 56,900 C) 58,200 D) 59,400 E) 61,800

Find Payments: $85{,}000 = P\,\ddot{a}_{\overline{18}|6.5\%} \rightarrow 85{,}000 = P(1.065)\,a_{\overline{18}|6.5\%} \rightarrow P = 7650.37$

Insurance Company's Transactions During first 8 yrs:



$X = B_8 = 85{,}000(1.07)^8 - 7650.37\,\ddot{s}_{\overline{8}|7\%} - 140\,s_{\overline{8}|7\%}(1.07)^{1/2}$

$= 60{,}574.3544$

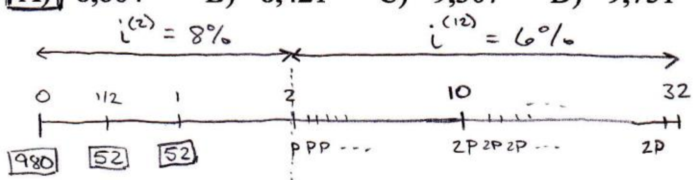
3. John took out a 2,020,000 construction loan, disbursed to him in three installments. The first installment of 980,000 is disbursed immediately, followed by two 520,000 installments at six month intervals.

The interest on the loan is calculated at a rate of 8% convertible semiannually and accumulated to the end of the second year. At that time, the loan and accumulated interest will be replaced by a 30-year mortgage at 6% convertible monthly.

The amount of the monthly mortgage payment for the first 8 years will be one-half of the payment for all later years. The first monthly mortgage payment is due exactly two years after the initial disbursement of the construction loan.

Calculate the amount of the 12th mortgage payment. [3.a-c #12]

$\boxed{\text{A)}}$ 8,864 B) 8,421 C) 9,307 D) 9,751 E) 10,194



12th payment = P

$\boxed{t=2} \rightarrow 980(1.04)^4 + 520(1.04)^3 + 520(1.04)^2 = 2P\ddot{a}_{\overline{360}|0.005} - P\ddot{a}_{\overline{96}|0.005}$ (using block payments. See 3e)

$2293.8227 = P\left[2(1.005)a_{\overline{360}|0.005} - (1.005)a_{\overline{96}|0.005}\right]$

$2293.8227 = P(258.7755)$

$P = 8.864$

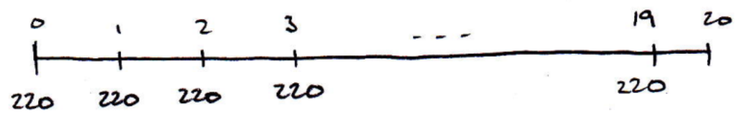
4. At the beginning of each year for 20 years $220 is deposited into a savings account.

At a simple annual interest rate of *i*%, the total amount in the account is $9020 at the end of 20 years.

To the nearest $5, what would be the total amount in the account at the end of 20 years if interest had been compounded at an effective annual interest rate of *i*%? [3.a-c #17]

$\boxed{\text{A)}}$ $13,860 B) $14,070 C) $14,280 D) $14,480 E) $14,690

Note: $1+2+3+\dots+n = \frac{n(n+1)}{2}$ (This is worth memorizing)



$$220(1+20i) + 220(1+19i) + \dots + 220(1+2i) + 220(1+i) = 9020$$

$$(1+20i) + (1+19i) + \dots + (1+2i) + (1+i) = 41$$

$$20 + i(1+2+3+\dots+19+20) = 41$$

$$i\,\frac{20(21)}{2} = 21$$

$$i = \frac{1}{10} = 0.1$$

Using compound interest: $220\,\ddot{s}_{\overline{20}|10\%} = 220(1.1)\frac{(1.1)^{20}-1}{0.1}$

$= 13,860.55$

5. Jeff and John shared equally in an inheritance.

Using his inheritance, John immediately bought a 11-year annuity-due with annual payments of 5500 each.

Jeff put his inheritance in an investment fund earning an annual effective interest rate of 5.1%. After 5 years, Jeff bought a 13-year annuity immediate with annual payment of Z.

The present value of both annuities was determined using an annual effective interest rate of 9.6%.

Calculate Z. [3.a-c #03]

$\boxed{\text{A)}}$ 7052 B) 6629 C) 6770 D) 6911 E) 7193

$X = 5500\,\ddot{a}_{\overline{11}|9.6\%} = 5500(1.096)\,a_{\overline{11}|9.6\%} = 39{,}883.73$

$39883.73(1.051)^5 = Z\,a_{\overline{13}|9.6\%} \rightarrow Z = 7051.67$