HW 2.3(a) Key

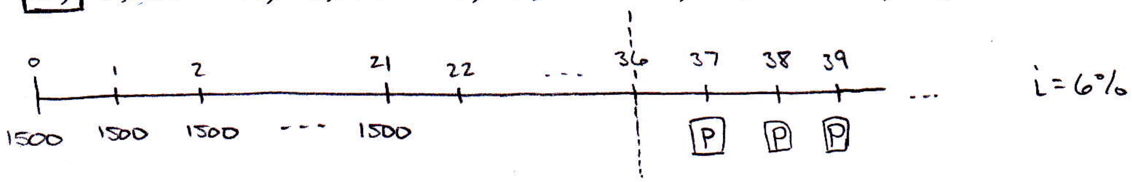
1. . Deposits of 1500 are placed into a fund at the beginning of each year for 22 years.

At the end of year 37, annual payments commence and continue forever.

Interest is at an effective annual rate of 6%.

Calculate the annual payment. [3.d-f #19]

$\boxed{\text{A)}}$ 9,360 B) 9,270 C) 9,450 D) 9,550 E) 9,640



$1500\,\ddot{s}_{\overline{22}|}(1.06)^{14} = P\,a_{\overline{\infty}|}$ $\rightarrow$ $P = \boxed{9359.29}$

2. Victor wants to purchase a perpetuity paying 120 per year with the first payment due at the end of year 16. He can purchase it by either:
(i) paying 140 per year at the end of each year for 15 years; or
(ii) paying *K* per year at the end of each year for the first 10 years and nothing for the next 5 years.
Calculate *K*. [3.d-f #21]

$\boxed{\text{A)}}$ 190 B) 165 C) 180 D) 205 E) 220

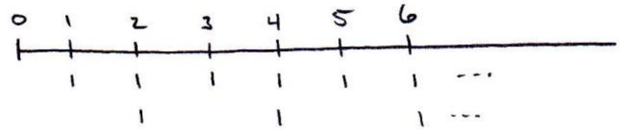
i) $140\,s_{\overline{15}|i} = 120\,a_{\overline{\infty}|i}$ $\xrightarrow{\text{(mult by } i)}$ $140\left[(1+i)^{15}-1\right] = 120$ $\rightarrow$ $i = 4.2133\%$

ii) $K\,s_{\overline{10}|i}(1+i)^5 = 120\,a_{\overline{\infty}|i}$ $\rightarrow$ $K(14.9044) = 2848.1445$

$K = \boxed{191.09}$

3. A perpetuity pays 60 at the end of each year, plus an additional 60 at the end of every second year. Assuming an annual effective rate of interest of 6%, find the present value of this annuity.

$\boxed{\text{A)}}$ 1,485.44 B) 1,455.73 C) 1,515.15 D) 1,544.85 E) 1,574.56



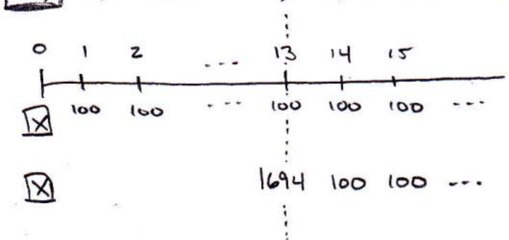
(1 yr rate) $i = 6\%$

(2 yr rate) $j = 12.36\%$

$$PV = 60\,a_{\overline{\infty}|i} + 60\,a_{\overline{\infty}|j} = \frac{60}{0.06} + \frac{60}{0.1236} = \boxed{1485.44}$$

4. The following two payment options each has a present value of X.
(i) 100 at the end of each year, forever, with the first payment due at $t = 1$.
(ii) A payment of 1694 at $t = 13$, followed by 100 at the end of each year, forever, with the first payment of 100 due at $t = 14$.
Find X.

$\boxed{\text{A)}}$ 2,325.58 B) 2,139.53 C) 2,186.05 D) 2,232.56 E) 2,279.07



$$100\,s_{\overline{13}|i} + 100\,a_{\overline{\infty}|i} = 1694 + 100\,a_{\overline{\infty}|i}$$

$$100\,s_{\overline{13}|i} = 1694 \rightarrow i = 4.3\%$$

$$X = 100\,a_{\overline{\infty}|i} = \frac{100}{0.043} = \boxed{2325.58}$$

5. At an annual effective rate of interest i, the present value of a perpetuity-immediate with payments of 80 is the same as the accumulated value of an n-year annuity immediate with payments of 120. At the same effective rate of interest, determine the present value of a payment of 300 occuring at the end of year n.

$\boxed{\text{A)}}$ 180 B) 164 C) 169 D) 175 E) 185

$$80\,a_{\overline{\infty}|i} = 120\,s_{\overline{n}|i}$$

$$\frac{80}{i} = 120\,\frac{(1+i)^n - 1}{i}$$

$$0.66667 = (1+i)^n - 1$$

$$(1+i)^n = 1.66667 \rightarrow v^n = 0.6$$

PV of 300 at $t = n$ $= 300\,v^n$

$= 300\,(0.6)$

$= \boxed{180}$