HW 2.3(b) Key

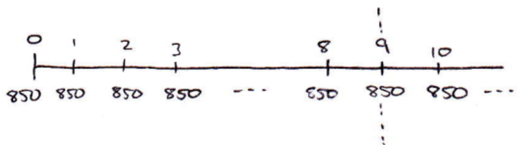
1. Ralph buys a perpetuity-due paying 850 annually. He deposits the payments into a savings account earning interest at an effective annual rate of 6%.

At the end of year 9, before receiving payment number 10, Ralph sells the perpetuity based on an effective annual interest rate of 6%.

Using the proceeds from the sale plus the money in the savings account, Ralph purchases an annuity due paying *X* per year for 16 years at an annual effective interest rate of 6%.

Calculate *X*. [3.d-f #16]

$\boxed{\text{A)}}$ 2,368 B) 2,226 C) 2,297 D) 2,439 E) 2,510



$$850\,\ddot{s}_{\overline{9}|6\%} + 850\,\ddot{a}_{\overline{\infty}|6\%}$$

$$\left[850\,\ddot{s}_{\overline{9}|6\%} + 850\,\ddot{a}_{\overline{\infty}|6\%} = X\,\ddot{a}_{\overline{16}|6\%}\right] \div (1.06)$$

$$850\,s_{\overline{9}|6\%} + 850\,a_{\overline{\infty}|6\%} = X\,a_{\overline{16}|6\%} \rightarrow \boxed{X = 2368.35}$$

2. The following three series of payments have the same present value of *P*:
(i) a perpetuity-immediate of 10 per year at an annual effective interest rate of *i*;
(ii) a 26-year annuity-immediate of *X* per year at an annual effective interest rate of 2*i*;
(iii) a 26-year annuity-due of 0.86957*X* per year at an annual effective interest rate of 2*i*.

Calculate *P*. [3.d-f #18]

$\boxed{\text{A)}}$ 133 B) 113 C) 120 D) 127 E) 140

i) $P = 10\,a_{\overline{\infty}|i}$

ii) $P = X\,a_{\overline{26}|2i}$

iii) $P = 0.86957X\,\ddot{a}_{\overline{26}|2i}$

(ii & iii) $X a_{\overline{26}|2i} = 0.86957X(1+2i)\,a_{\overline{26}|2i}$

$$1 = 0.86957(1+2i)$$

$$i = 0.075$$

(i) $P = 10/0.075 = \boxed{133.34}$

3. The death benefit on a life insurance policy can be paid in any of the following ways, each of which has the same present value as the death benefit:

(i) a perpetuity of 180 at the end of each month;
(ii) 503.49 at the end of each month for n years; and
(iii) a payment of 25,234 at the end of n years.

P = Present Value
j = monthly rate

Calculate the amount of the death benefit. [3.d-f #20]

$\boxed{\text{A)}}$ 16,200 B) 14,600 C) 15,400 D) 17,000 E) 17,800

i) $P = 180\, a_{\overline{\infty}|j}$

ii) $P = 503.49\, a_{\overline{12n}|j}$

iii) $P = 25{,}234\, v_j^{12n}$

(i & ii) $\dfrac{180}{j} = 503.49\, \dfrac{1 - v^{12n}}{j}$

$180 = 503.49\,(1 - v^{12n})$

$v^{12n} = 0.6425$

(iii) $P = 25{,}234\,(0.6425)$

$= \boxed{16{,}212.73}$

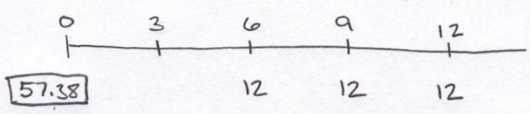
4. At an annual effective interest rate of i, $i > 0$, the present value of a perpetuity of 12 at the end of each 3-year period, with the first payment at the end of year 6, is 57.38.

At the same annual effective rate of i, the present value of a perpetuity-immediate paying 2 at the end of each 4-month period is X.

Calculate X. [3.d-f #03]

$\boxed{\text{A)}}$ 109.1 B) 114.6 C) 120.0 D) 125.5 E) 130.9

i = effective annual rate j = effective 3 yr rate



$57.38 = 12\, a_{\overline{\infty}|j}\, v_j = \dfrac{12}{j}\,\dfrac{1}{1-j}$

$57.38 j^2 + 57.38 j - 12 = 0$ (4-month rate)

$j = 0.17759 \rightarrow i = 5.6\% \rightarrow k = 0.01833$

$X = 2\, a_{\overline{\infty}|k} = \dfrac{2}{k} = \boxed{109.1}$

5. A loan is to be repaid by annual payments continuing forever, the first one due one year after the loan is made. Find the amount of the loan if the payments are 1, 2, 3, 4, 1, 2, 3, 4, ... assuming an annual effective interest rate of 8%. [3.d-f #07]

$\boxed{\text{A)}}$ At least 30, but less than 31

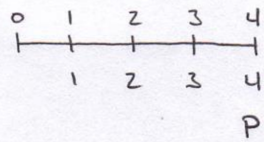
B) At least 31, but less than 32

C) At least 32, but less than 33

D) At least 33, but less than 34

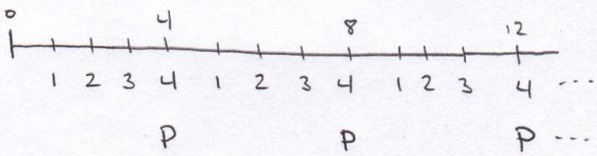
E) At least 34, but less than 35

This solution uses fusion, which is covered in Section 4.b.



$$P = 1(1.08)^3 + 2(1.08)^2 + 3(1.08) + 4 = 10.8325$$

So, payments of 1, 2, 3, 4 at the end of consecutive years can be replaced by a single pmt of 10.8325 at the end of the 4th yr.



So, these two payment options are equivalent.

(4 yr rate) $k = 0.36049$

$$PV = P\,a_{\overline{\infty}|k} = \frac{10.8325}{0.36049} = \boxed{30.05}$$