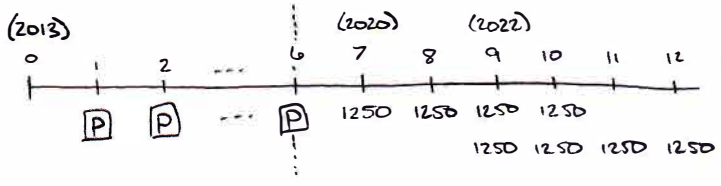
HW 2.4 Key

1. Ms. Smith has two grandchildren, Adam and Evelyn. Adam will be enrolling in college on September 1, 2020, and Evelyn will be enrolling in college on September 1, 2022. Ms. Smith wishes to give both Adam and Evelyn 1250 at the beginning of each of their four years of college.

   Ms. Smith will fund these payments by making 6 level annual deposits of P into an account earning an effective interest rate of 8%, with the first deposit on September 1, 2014.

   Determine the value of P. [3.d-f #08]

   A) At least 1,000, but less than 1,050
   B) At least 1,050, but less than 1,100
   C) At least 1,100, but less than 1,150
   D) At least 1,150, but less than 1,200
   E) At least 1,200, but less than 1,250



$i = 8\%$

$Rs_{\overline{6}|} = 1250\,a_{\overline{4}|} + 1250\,a_{\overline{4}|}v^2$

$Rs_{\overline{6}|} = 1250\,a_{\overline{4}|}(1+v^2)$

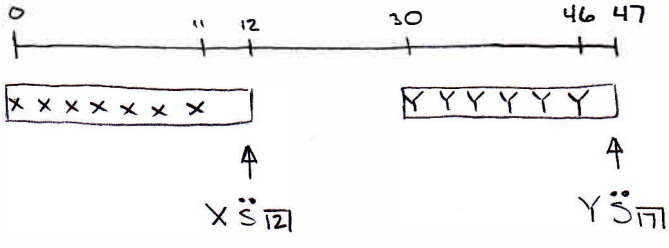
$R = \boxed{1048.22}$

2. Eloise plans to accumulate 80,000 at the end of 47 years. She makes the following deposits:
   (i) X at the beginning of years 1 - 12;
   (ii) No deposits at the beginning of years 13 - 30;
   (iii) Y at the beginning of years 31 - 47.

   The annual effective interest rate is 4%. You are given that $X - Y = 150$. $\rightarrow$ $X = 150 + Y$

   Calculate Y. [3.d-f #11]

   A) 820 B) 853 C) 885 D) 918 E) 951



$80{,}000 = X\ddot{s}_{\overline{12}|}(1.04)^{35} + Y\ddot{s}_{\overline{17}|}$

$80{,}000 = (150+Y)(61.666) + Y(24.65)$

$80{,}000 = 9249 + 86.31Y$

$Y = \boxed{819.73}$

3. A 36 year annuity pays 6 during years 1-9, 12 during years 10-18, 3 during years 19-27, and 10 during years 28-36. Payments occur at the end of the year. Which of the following expressions represents the present value of this annuity?

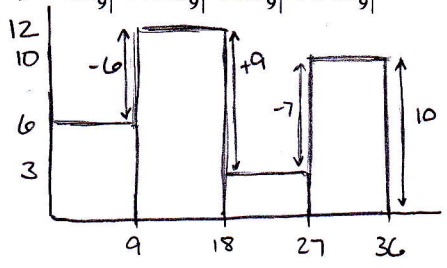
A) $10a_{\overline{36}|} - 7a_{\overline{27}|} + 9a_{\overline{18}|} - 6a_{\overline{9}|}$

B) $6a_{\overline{9}|} + 12a_{\overline{18}|} + 3a_{\overline{27}|} + 10a_{\overline{36}|}$

C) $6a_{\overline{9}|} + 12a_{\overline{9}|} + 3a_{\overline{9}|} + 10a_{\overline{9}|}$

D) $6a_{\overline{9}|} + 6a_{\overline{18}|} - 9a_{\overline{27}|} + 7a_{\overline{36}|}$

E) $10a_{\overline{9}|} - 7a_{\overline{9}|} + 9a_{\overline{9}|} - 6a_{\overline{9}|}$



$10a_{\overline{36}|} - 7a_{\overline{27}|} + 9a_{\overline{18}|} - 6a_{\overline{9}|}$

4. Annuities X and Y provide the following payments:

End of Year	Annuity X	Annuity Y
1 - 10	8	K
11 - 20	3	0
21 - 30	4	K

Annuities X and Y have equal present values at an effective annual interest rate i such that $v^{10} = 0.07$.

Determine K. [3.d-f #27]

A) 8.19 B) 7.78 C) 8.6 D) 9.01 E) 9.42

mult. by i

$4a_{\overline{30}|} - a_{\overline{20}|} + 5a_{\overline{10}|} = Ka_{\overline{30}|} - Ka_{\overline{20}|} + Ka_{\overline{10}|}$

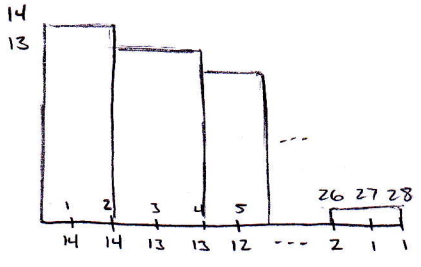
$4(1-v^{30}) - (1-v^{20}) + 5(1-v^{10}) = K(1-v^{30}) - K(1-v^{20}) + K(1-v^{10})$

$7.6535 = K(0.934557) \rightarrow K = \boxed{8.19}$

5. An annuity-immediate pays 14 at the ends of years 1 and 2, 13 at the ends of years 3 and 4, etc., with payments decreasing by 1 every second year until nothing is paid. The effective annual interest rate is 6%.

Calculate the present value of this annuity-immediate. [3.d-f #23]

A) 125 B) 131 C) 137 D) 144 E) 150



$PV = a_{\overline{28}|} + a_{\overline{26}|} + a_{\overline{24}|} + \ldots + a_{\overline{4}|} + a_{\overline{2}|}$

$= \frac{1}{i}\left[(1-v^{28}) + (1-v^{26}) + \ldots + (1-v^4) + (1-v^2)\right]$

$= \frac{1}{i}\left[14 - (v^2 + v^4 + v^6 + \ldots + v^{26} + v^{28})\right]$

$= \frac{1}{i}\left[14 - \frac{v^2(1-v^{28})}{1-v^2}\right] = \boxed{124.87}$