HW 2.5 (a) Key

1. Jerry will make deposits of 600 at the end of each quarter for 8 years. At the end of 14 years, Jerry will use the fund to make annual payments of Y at the beginning of each year for 6 years, after which the fund is exhausted. The annual effective rate of interest is 5.5%. Determine Y. [4.a-c #02]

[A] 6,229 B) 6,104 C) 6,166 D) 6,291 E) 6,353

$i = 5.5\%$ $\quad j = 1.3475\%$ (quarterly)

$600\, s_{\overline{32}|j}\,(1.055)^6 = Y\, \ddot{a}_{\overline{6}|i}$

$\boxed{Y = 6228.69}$

2. A perpetuity paying 1 at the beginning of each 6-month period has a present value of 100. A second perpetuity pays X at the beginning of every 3 years. Assuming the same annual effective interest rate, the two present values are equal. Determine X. [4.a-c #03]

[A] 5.9 B) 6 C) 6.1 D) 6.2 E) 6.3

j = 6 month rate $\quad k$ = 3 yr rate

$100 = \ddot{a}_{\infty|j} = 1 + \frac{1}{j} \rightarrow j = 0.010101 \rightarrow k = 0.062157$

$100 = X\, \ddot{a}_{\infty|k} = X\left(1 + \frac{1}{k}\right) \rightarrow X = \boxed{5.85}$

3. A 28-year annuity pays 220 every other year beginning at the end of the second year, with additional payments of 360 at the end of years 5, 13, and 21. The effective annual interest rate is 10%. Calculate the present value of the annuity. [4.a-c #19]

[A] 1,350 B) 1,320 C) 1,390 D) 1,420 E) 1,450

$i = 10\%$ $\quad$ (2 yr rate) $j = 21\%$

$PV = 220\, a_{\overline{14}|j} + 360\left(v^5 + v^{13} + v^{21}\right)$

$= \boxed{1351.43}$

4. You are given:
(i) the present value of a $4n$-year annuity-immediate of 1 at the end of every year is 12.594;
(ii) the present value of a $4n$-year annuity-immediate of 1 at the end of every second year is 6.070;
(iii) the present value of a $4n$-year annuity-immediate of 1 at the end of every third year is X;
Calculate X. [4.a-c #20]

A) 3.9 B) 3.51 C) 3.7 D) 4.09 E) 4.29

i) $12.594 = a_{\overline{4n}|}$

ii) $6.070 = \frac{1}{s_{\overline{2}|}} a_{\overline{4n}|}$ ← using fission

iii) $X = \frac{1}{s_{\overline{3}|}} a_{\overline{4n}|}$ ←

(i & ii) $s_{\overline{2}|} = 2.07479$

$i = 7.4790\%$

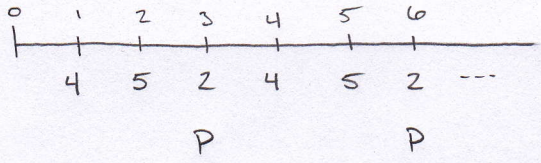
$s_{\overline{3}|} = 3.23$

(i & iii) $X = \frac{1}{3.23}(12.594)$

$= \boxed{3.899}$

5. You are given a perpetuity, with annual payments as follows:
(i) Payments of 4 at the end of the first year and every three years thereafter.
(ii) Payments of 5 at the end of the second year and every three years thereafter.
(iii) Payments of 2 at the end of the third year and every three years thereafter.
The interest is 7% convertible semiannually. Calculate the present value of this perpetuity. [4.a-c #22]

A) 52 B) 42 C) 47 D) 57 E) 63



(nominal) $i^{(2)} = 7\%$

(semi-annual) $j = 3.5\%$

(effective) $i = 7.1225\%$

(3 year) $k = 22.9255\%$

$P = 4(1+i)^2 + 5(1+i)^1 + 2$

$= 11.946217$

$PV = P a_{\overline{\infty}|k} = \frac{P}{k}$

$= \boxed{52.109}$