HW 2.5 (b) Key

1. Kathryn deposits 180 into an account at the beginning of each 5 year period for 60 years. The account credits interest at an annual effective interest rate of i. The accumulated amount in the account at the end of 60 years is X, which is 6 times the accumulated amount in the account at the end of 30 years. Calculate X. [4.a-c #01]

$\boxed{\text{A)}}$ 18,360 B) 17,445 C) 19,280 D) 20,200 E) 21,115

i = annual rate j = 5 yr rate

$$X = AV_{60} = 6\,AV_{30}$$

$$180\,\ddot{s}_{\overline{12}|j} = 6(180)\,\ddot{s}_{\overline{6}|j}$$

$$s_{\overline{12}|j} = 6\,s_{\overline{6}|j}$$

$$\frac{s_{\overline{12}|j}}{s_{\overline{6}|j}} = 6$$

$$\rightarrow \; 1 + (1+j)^6 = 6 \;\rightarrow\; j = 0.307666 \quad (*)$$

$$X = 180\,\ddot{s}_{\overline{12}|j} = \boxed{18{,}361.45}$$

(*) In this step, we used the rule $\frac{s_{\overline{2n}|}}{s_{\overline{n}|}} = 1 + (1+i)^n$, which is similar to $\frac{a_{\overline{2n}|}}{a_{\overline{n}|}} = 1 + v^n$. We could've also used the quad. formula to find j.

2. The present value of a perpetuity is \$12. The perpetuity pays \$1 at the end of every 3 years, with the first payment due immediately. Cheryl will make 10 payments of \$100 each. The first payment is due one year from now, with successive payment due every 2 years thereafter. Determine the present value of Cheryl's payments at the same annual effective interest rate used to determine the present value of the perpetuity described above. [4.a-c #05]

$\boxed{\text{A)}}$ At least 750, but less than 775
B) At least 700, but less than 725
C) At least 725, but less than 750
D) At least 775, but less than 800
E) At least 800, but less than 825

i = annual rate j = 2 yr rate k = 3 yr rate

$$12 = \ddot{a}_{\overline{\infty}|k} = 1 + \frac{1}{k} \;\rightarrow\; k = 0.090909 \;\rightarrow\; j = 0.059723$$

$$i = 0.029428$$

$$PV = 100\,\ddot{a}_{\overline{10}|j}\,(1+i)^{-1} = \boxed{758.66}$$

3. Norma receives $700,000 from a life insurance policy with which she purchases an annuity-certain. The annuity will pay 16 equal annual installments, with the first payment made immediately. On the day she receives payment number 5 she is offered, in lieu of the future annual payments, a new payment scheme:
(i) 8 annual payments of $30,000, beginning in one year, followed by a monthly perpetuity of *X*.
(ii) The first monthly perpetuity payment would occur one month after the last annual payment of $30,000.
The effective annual rate of interest is 10% for the entire time period. Determine the value of *X*. [4.a-c #06]

$\boxed{\text{A)}}$ At least 6,200, but less than 6,300
B) At least 6,000, but less than 6,100
C) At least 6,100, but less than 6,200
D) At least 6,300, but less than 6,400
E) At least 6,400, but less than 6,500

$700 = P\ddot{a}_{\overline{16}|10\%} \rightarrow P = 81.3378$

Payment 5 received at $t=4$; 11 payments left

$B_4 = Pa_{\overline{11}|10\%} = 528.294$

$528.294 = 30a_{\overline{8}|10\%} + Xa_{\overline{\infty}|j}(1.1)^{-8}$ $\quad$ monthy $(j = 0.007974)$

$X = 6.295$ $\quad \boxed{6295}$

4. You currently have 1400 in an account which pays a nominal rate of interest of 12% compounded quarterly. You plan to deposit 100 every two months with the first deposit one month from now. What will be the value of the account one month after deposit number 14? [4.a-c #24]

$\boxed{\text{A)}}$ At least 3,440, but less than 3,460
B) At least 3,380, but less than 3,400
C) At least 3,400, but less than 3,420
D) At least 3,420, but less than 3,440
E) At least 3,460, but less than 3,480

$i^{(4)} = 12\%$ $\quad j = 3\%$ (quarterly) $\quad k =$ (2 month)

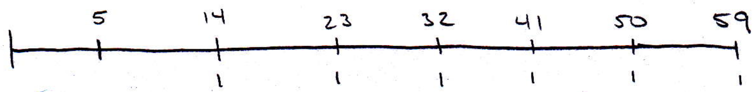
0 1 3 5 ... 25 27 28
1400 100 100 100 100 100

$AV = 1400(1+k)^{14} + 100s_{\overline{14}|k}(1+k)^{1/2}$

$= \boxed{3456.92}$

5. Determine the present value of 1 payable at the end of years 14, 23, 32, 41, 50, and 59. [4.a-c #26]

$\boxed{\text{A)}}$ $\dfrac{a_{\overline{59}|} - a_{\overline{5}|}}{s_{\overline{9}|}}$ B) $\dfrac{a_{\overline{60}|} - a_{\overline{6}|}}{s_{\overline{9}|}}$ C) $\dfrac{a_{\overline{59}|} - a_{\overline{5}|}}{a_{\overline{9}|}}$ D) $\dfrac{a_{\overline{59}|} - a_{\overline{5}|}}{s_{\overline{8}|} - a_{\overline{1}|}}$ E) $\dfrac{a_{\overline{59}|} - a_{\overline{5}|}}{s_{\overline{8}|} + a_{\overline{1}|}}$



Payments occur every 9 years. We will replace with annual payments of P every year, with first payment at t = 6.

$P s_{\overline{9}|} = 1 \rightarrow P = \dfrac{1}{s_{\overline{9}|}}$

Using block payments:

$PV = P s_{\overline{59}|} - P s_{\overline{5}|} = \dfrac{1}{s_{\overline{9}|}} s_{\overline{59}|} - \dfrac{1}{s_{\overline{9}|}} s_{\overline{5}|}$

$= \boxed{\dfrac{s_{\overline{59}|} - s_{\overline{5}|}}{s_{\overline{9}|}}}$