

HW 2.6 (a) Key

1. Olga buys a 4-year increasing annuity for X . Olga will receive 4 at the end of the first month, 8 at the end of the second month, and for each month thereafter the payment increases by 4. The nominal interest rate is 8% convertible quarterly. Calculate X . [4.h-i #02]

(A) 3,810 B) 3,890 C) 3,960 D) 4,040 E) 4,120

$$i^{(4)} = 8\% \quad j = 2\% \text{ (quarterly)} \quad k = 0.66227\%$$

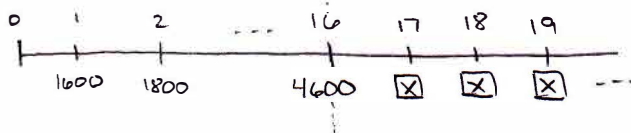
$$X = 4(Ia)_{\overline{48}|k} = 4 \frac{\ddot{a}_{\overline{48}|k} - 48v_k^{48}}{k}$$

$$= \boxed{3810.91}$$

2. A scholarship fund is to be established by making deposits at the end of each year for 16 years. The first deposit is 1600, and each subsequent deposit increases by 200.

Beginning at time $t = 17$, the fund will pay one scholarship of amount X annually into perpetuity. Assuming $i = 0.05$, what is the largest value of X that the fund can provide? [4.h-i #07]

- (A) At least 3,250, but less than 3,500 D) At least 4,000, but less than 4,250
 B) At least 3,500, but less than 3,750 E) At least 4,250, but less than 4,500
 C) At least 3,750, but less than 4,000



$$1600 s_{\overline{16}|0.05} + 200 \frac{s_{\overline{16}|0.05} - 16}{0.05} = \frac{X}{0.05} \rightarrow X = \boxed{3424.10}$$

3. Francois purchases a 7-year annuity-immediate with annual payments of $18X$. Jacques purchases a 7-year decreasing annuity-immediate which also makes annual payments. The payment at the end of year 1 is equal to 100. At the end of year 2, and at the end of each year through year 7, each subsequent payment is reduced over what was paid in the previous year by an amount equal to X . At an annual effective interest rate of 8%, both annuities have the same present value. Calculate X . [4.h-i #09]

(A) 4.83 B) 4.95 C) 5.07 D) 5.19 E) 5.32

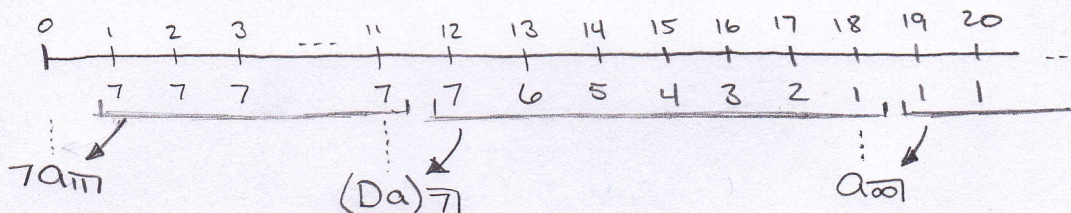
$$18X a_{\overline{7}|8\%} = 100 a_{\overline{7}|8\%} - X \frac{a_{\overline{7}|8\%} - 7v^7}{0.08}$$

$$107.739X = 520.637$$

$$X = \boxed{4.83}$$

4. You are given an annuity-immediate paying 7 for 12 years and then decreasing by 1 per year for 6 years and paying 1 per year thereafter, forever. The annual effective rate of interest is 10%. Calculate the present value of this annuity. [4.h-i #36]

☒ A) 55 B) 53 C) 56 D) 58 E) 60



$$PV = 7a_{\overline{7}|} + (Da)_{\overline{7}|} v^{11} + a_{\infty} v^{18}$$

$$= \boxed{54.74}$$

5. Two annuities have the same present value. The first annuity is a decreasing annual annuity. The first payment is 900, due one year from today. Subsequent annual payments decrease by 90 per year. The interest rate is 10% compounded annually. The second annuity provides payments of K per month for 10 years. The first payment is due one month from today. The interest rate is 10% compounded annually. What is K ? [4.h-i #48]

☒ A) At least 44, but less than 46
 B) At least 46, but less than 48
 C) At least 48, but less than 50

D) At least 50, but less than 52
 E) At least 52, but less than 54

$$i = 10\% \text{ (annual)} \quad j = 0.797414\% \text{ (monthly)}$$

$$90(Da)_{\overline{10}|i} = K a_{\overline{120}|j}$$

$$K = \boxed{45.03}$$