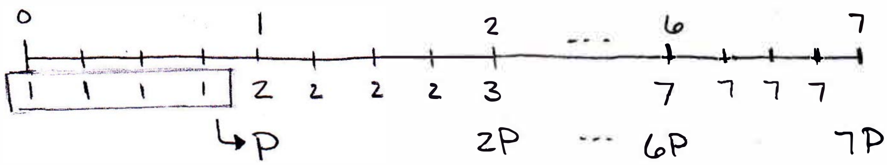
HW 2.6 (b) Key

1. Scott makes deposits at the beginning of each quarter of 7 years. The size of the quarterly deposits are 1 during year 1, 2 during year 2, 3 during year 3, and so on.

   One quarter after the last deposit, Scott withdraws the accumulated value of the fund and uses it to buy a perpetuity-immediate with level payments of X at the end of each year. All calculations assume a nominal interest rate of 12% per annum compounded quarterly. Calculate X. [4.h-i #05]

   $\boxed{\text{A)}}$ 19.6 B) 18.1 C) 18.6 D) 19.1 E) 20.1

$i^{(12)} = 12\%$ $j = 3\%$ (quarterly) $i = 12.55\%$ (annual effective)

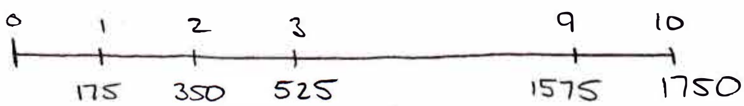


$P = \ddot{s}_{\overline{4}|j} = 4.3091$ (using fusion)

$P(Is)_{\overline{7}|i} = X a_{\overline{\infty}|i} \rightarrow X = \boxed{19.68}$

2. Bill deposits money into a bank account at the end of each year. Bill's deposit in year t is equal to $175t$. The bank credits interest at an annual effective rate of i. The amount of interest earned in Bill's account during year 10 is equal to 1050. Calculate i. [4.h-i #15]

   A) 10.08% B) 9.07% C) 9.58% D) 10.59% E) 11.09%



$B_9 = 175(Is)_{\overline{9}|}$

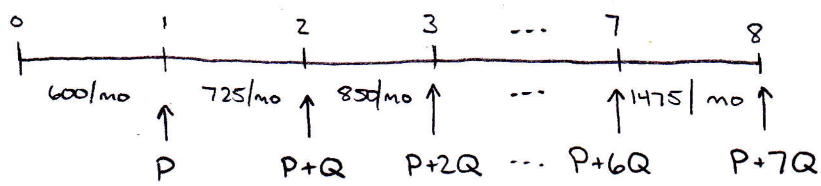
$I_{10} = i \cdot B_9 = 175\left[\ddot{s}_{\overline{9}|} - 9\right] = 1050$

$\ddot{s}_{\overline{9}|} = 15 \rightarrow s_{\overline{10}|} - 1 = 15 \rightarrow s_{\overline{10}|} = 16$

$i = \boxed{10.08\%}$

3. Joan has won a lottery that pays 600 per month in the first year, 725 per month in the second year, 850 per month in the third year, etc. Payments are made at the end of each month for 8 years. Using an effective interest rate of 5% per annum, calculate the present value of this prize. [4.h-i #34]

$\boxed{\text{A)}}$ 80,000 B) 72,500 C) 87,000 D) 94,000 E) 101,500



$i = 5\%$

$j = 0.4074\%$ (monthly)

$$P = 600\, s_{\overline{12|}j} = 7363.5465$$

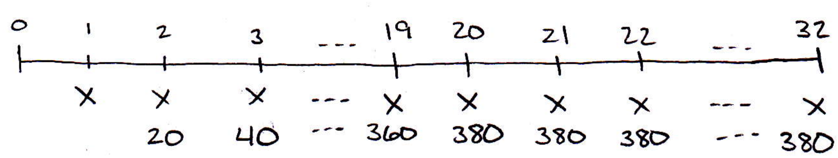
$$Q = 125\, s_{\overline{12|}j} = 1534.0722$$

$$PV = P a_{\overline{8|}i} + Q\, \frac{a_{\overline{8|}i} - 8v^8}{0.05}$$

$$= \boxed{79{,}761.60}$$

4. Gloria borrows 5,000 to be repaid over 32 years. You are given:
   (i) Her first payment is X at the end of year 1.
   (ii) Her payments increase at the rate of 20 per year for the next 19 years and remain level for the following 12 years.
   (iii) The effect rate of interest is 7% per annum.
   Calculate X. [4.h-i #35]

A) 210 B) 155 C) 175 D) 190 E) 230



$$5000 = X a_{\overline{32|}7\%} + 20 (Ia)_{\overline{19|}7\%} \cdot v + 380\, a_{\overline{12|}7\%} \cdot v^{20}$$

$$12.6466\, X = 2669.85$$

$$X = \boxed{211.11}$$

5. Annuity A pays 1 at the end of each *m*th of a year during year 1, 2 at the end of each *m*th of a year during year 2, and so on, for 8 years.

Annuity B pays $1/m$ at time $1/m$, $2/m$ at time $2/m$, and so on, for 8 years.

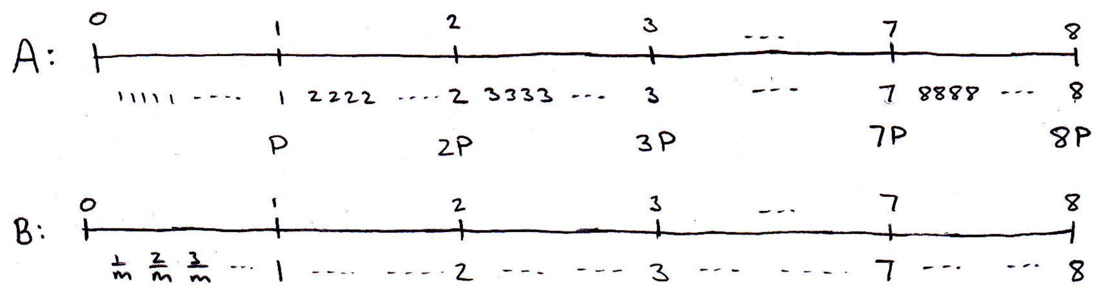
The present value of annuity A is 1.1112 times the present value of annuity B at an annual effective interest rate of 5.7%.

Find $d^{(m)}$.

$\boxed{\text{A)}}$ 5.52% B) 5.41% C) 5.63% D) 5.74% E) 5.85%

j = m-thly interest rate.

Notice: $1 - \frac{d^{(m)}}{m} = \frac{1}{1+j} \rightarrow d^{(m)} = m\left(1 - \frac{1}{1+j}\right) = \frac{mj}{1+j}$



$(P = s_{\overline{m}|j})$

$$s_{\overline{m}|j}(Ia)_{\overline{8}|i} = 1.1112\left(\frac{1}{m}\right)(Ia)_{\overline{8m}|j}$$

$$\cancel{i} = \quad \frac{(1+j)^m - 1}{\cancel{j}} \cdot \frac{\ddot{a}_{\overline{8}|i} - 8v^8}{\cancel{i}} = 1.1112\left(\frac{1}{m}\right)\frac{\ddot{a}_{\overline{8m}|j} - 8m v_j^{8m}}{\cancel{j}}$$

$$\ddot{a}_{\overline{8}|i} - 8v^8 = 1.1112\left(\frac{1}{m}\right)\left[(1+j)\frac{1-v_j^{8m}}{j} - 8m v_j^{8m}\right]$$

$$\ddot{a}_{\overline{8}|i} - 8v^8 = 1.1112\left(\frac{1+j}{jm}\right)(1-v^8) - 8(1.1112)v^8$$

$$\ddot{a}_{\overline{8}|i} - 8v^8 + 8(1.1112)v^8 = 1.1112\left(\frac{1}{d^{(m)}}\right)(1-v^8)$$

$$d^{(m)} = \frac{1.1112(1-v^8)}{\ddot{a}_{\overline{8}|i} - 8v^8 + 8(1.1112)v^8} = \boxed{0.05518}$$

That was fun, right?

See next page for an alternate solution.

$$\text{PV of } A = m(Ia)^{(m)}_{\overline{8}|i}$$

$$\text{PV of } B = m(I^{(m)}a)^{(m)}_{\overline{8}|i}$$

$$\cancel{m}(Ia)^{(m)}_{\overline{8}|i} = 1.1112\ \cancel{m}(I^{(m)}a)^{(m)}_{\overline{8}|i}$$

$$\frac{\ddot{a}_{\overline{8}|i} - 8v^8}{\cancel{i^{(m)}}} = 1.1112\ \frac{\ddot{a}^{(m)}_{\overline{8}|i} - 8v^8}{\cancel{i^{(m)}}}$$

$$\ddot{a}_{\overline{8}|i} - 8v^8 = 1.1112\left(\ddot{a}^{(m)}_{\overline{8}|i} - 8v^8\right)$$

$$1.507995 = 1.1112\left[\frac{1-v^8}{d^{(m)}} - 5.1344\right]$$

$$1.35709 = \frac{0.358199}{d^{(m)}} - 5.1344$$

$$d^{(m)} = \boxed{0.05518}$$