HW 2.6 (d) Key

1. Brian buys a 22-year decreasing annuity-immediate with annual payments of 22, 21, 20, ..., 1. On the same date, Jenny buys a perpetuity-immediate with annual payments. For the first 23 years, payments are 1, 2, 3, ..., 23. After year 23, payments remain constant at 23. At an annual effective interest rate of i, both annuities have a present value of X. Calculate X. [4.h-i #20]

$\boxed{\text{A)}}$ 150.6 B) 132.5 C) 138.5 D) 144.5 E) 156.6

$X = (Da)_{\overline{22}|}$

$X = \dfrac{\ddot{a}_{\overline{23}|}}{i}$

$(Da)_{\overline{22}|} = \dfrac{\ddot{a}_{\overline{23}|}}{i}$

$22 - a_{\overline{22}|} = \ddot{a}_{\overline{23}|}$

$22 - a_{\overline{22}|} = a_{\overline{22}|} + 1$

$a_{\overline{22}|} = 10.5 \rightarrow i = 7.6375\%$

$X = \dfrac{\ddot{a}_{\overline{23}|}}{i} = \boxed{150.57}$

2. The first payment of a perpetuity-immediate is 62. Subsequent payments decrease by 1 per year until they reach a level of k. Payments remain constant at k thereafter. The present value of the perpetuity is equal to the present value of a perpetuity-immediate paying 53.1 each year. The annual effective interest rate is 11%. Calculate k. [4.h-i #21]

$\boxed{\text{A)}}$ 25 B) 27 C) 30 D) 32 E) 34

0 1 2 3 ... 63-k

62 61 60 ... k k k

$63-1$ $63-2$ $63-3$ $63-(63-k)$

0 1 2 3

53.1 53.1 53.1 ...

$i = 11\%$

$53.1\, a_{\overline{\infty}|} = 63 a_{\overline{\infty}|} - \dfrac{\ddot{a}_{\overline{63-k}|}}{i}$

$\ddot{a}_{\overline{63-k}|} = 9.9$

$a_{\overline{63-k}|} = 8.9189$

$63-k = 38 \rightarrow \boxed{k = 25}$

3. Perpetuity X has annual payments of 3, 6, 9, ... at the end of each year. Perpetuity Y has annual payments of q, q, $2q$, $2q$, $3q$, $3q$ ... at the end of each year. The present value of X is equal to the present value of Y at an annual effective interest rate of 8.5%. Calculate q. [4.h-i #23]

$\boxed{\text{A)}}$ 5.76 B) 3.23 C) 3.86 D) 4.5 E) 5.13

3 6 9 12 15 18

q q 2q 2q 3q 3q

P 2P 3P

(fusion) $\rightarrow P = q\, s_{\overline{2}|i} = 2.085q$

$i = 8.5\%$ $j = 17.7225\%$ (2 yr)

$3(Ia)_{\overline{\infty}|i} = 2.085q\,(Ia)_{\overline{\infty}|j}$

$3\left(\frac{1}{i} + \frac{1}{i^2}\right) = 2.085q\left(\frac{1}{j} + \frac{1}{j^2}\right)$

$\boxed{q = 5.76}$

4. You are given:
   (i) The present value of a perpetuity of 120 per year, the first payment at the end of *n* years, is 206.
   (ii) The present value of an increasing perpetuity of 120, 240, 360, etc., the first payment at the end of *n* years, is 1,751.
   (iii) The annual effective interest rate is *i*.
   Calculate *i*. [4.h-i #24]

$\boxed{\text{A)}}$ 13.3% B) 12.1% C) 12.5% D) 12.9% E) 13.7%

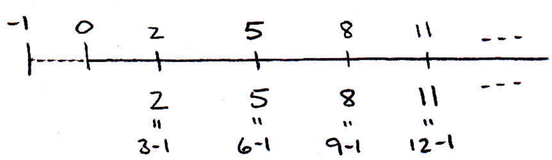
i) $206 = 120\, a_{\overline{\infty}|} \cdot v^{n-1} = 120\left(\frac{1}{i}\right) v^{n-1}$

ii) $1751 = 120\, (Ia)_{\overline{\infty}|} \cdot v^{n-1} = 120\left(\frac{1}{i} + \frac{1}{i^2}\right) v^{n-1}$

(Divide ii by i) $8.5 = 1 + \frac{1}{i} \rightarrow \boxed{i = 0.1333}$

5. A triennial perpetuity pays 2 at the end of the second year; 5 at the end of the fifth year; 8 at the end of the eighth year; 11 at the end of the eleventh year; and so on. The effective annual interest rate is 9.5%. Determine the present value of this perpetuity. [4.h-i #26]

$\boxed{\text{A)}}$ At least 40, but less than 42
B) At least 34, but less than 36
C) At least 36, but less than 38
D) At least 38, but less than 40
E) At least 42, but less than 44



$i = 9.5\%$

$j = 31.2932\%$

$PV = \left[3(Ia)_{\overline{\infty}|j} - a_{\overline{\infty}|j}\right](1+i)$

$= \left[3\left(\frac{1}{j} + \frac{1}{j^2}\right) - \frac{1}{j}\right](1+i)$

$= \boxed{40.54}$