HW 2.7 (a) Key

1. An annuity provides for 14 annual payments. The first payment is 80, paid at the end of the first year, and each subsequent payment is 3% more than the one preceding it. Calculate the present value of this annuity if $i = 0.04$. [4.j #05]

A) At least 1,010, but less than 1,020
B) At least 1,000, but less than 1,010
C) At least 1,020, but less than 1,030
D) At least 1,030, but less than 1,040
E) At least 1,040, but less than 1,050

$k = 3\%$ $\quad i = 4\%$ $\quad i' = \frac{0.04 - 0.03}{1.03} = 0.0097087$

$PV = \frac{80}{1.03} a_{\overline{14}|i'} = \boxed{1012.14}$

2. Jeff and Jason each spend X dollars to purchase an annuity. Jeff buys a perpetuity-immediate, which makes annual payments of 22. Jason buys a 18-year annuity-immediate, also with annual payments. The first payment is 39, with each subsequent payment $k\%$ larger than the previous year's payment. Both annuities use an annual effective interest rate of $k\%$. Calculate k. [4.j #08]

A) 3.24 B) 2.85 C) 3.04 D) 3.43 E) 3.62

Jeff: $X = 22 a_{\overline{\infty}|k}$ $\qquad 22\left(\frac{1}{k}\right) = \frac{39}{1+k}(18)$

Jason: $i = k$ $\quad i' = \frac{i-k}{1+k} = 0$ $\qquad 22 + 22k = 702k$

$X = \frac{39}{1+k} a_{\overline{18}|i'} = \frac{39}{1+k}(18)$ $\qquad k = \boxed{3.235\%}$

3. Mary is to receive an annuity with 18 annual payments. The first payment of 2400 is due immediately and each successive payment is 4% less than the payment for the preceding year. Interest is 7% compounded annually. Determine the present value of the annuity. [4.j #10]

A) At least 20,000, but less than 20,100
B) At least 19,600, but less than 19,700
C) At least 19,700, but less than 19,800
D) At least 19,800, but less than 19,900
E) At least 19,900, but less than 20,000

$k = -4\%$ $\quad i = 7\%$ $\quad i' = \frac{0.07 + 0.04}{1 - 0.04} = 11.4583\%$

$PV = \frac{2400}{0.96} a_{\overline{18}|i'} (1.07) = \boxed{20,033}$

4. An annuity provides for 28 annual payments. The first payment of 180 is made immediately and the remaining payments increase by 5% per annum. Interest is calculated at 9.9% per annum. Calculate the present value of this annuity. [4.j #17]

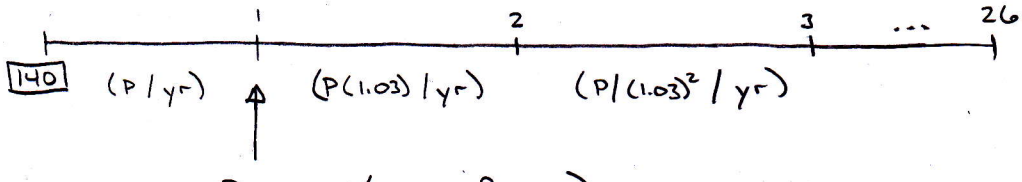
A) 2,911 B) 3,086 C) 3,261 D) 3,435 E) 3,610

$k = 5\%$ $\quad i = 9.9\%$ $\quad i' = \dfrac{0.099 - 0.05}{1.05} = 0.04667$

$PV = \dfrac{180}{1.05}\, a_{\overline{28}|i'}\,(1.099) = \boxed{2911.4}$

5. A loan of 140,000 will be repaid by payments at the end of each month over 26 years. Payments for a given year are level and are 3% greater than those for the previous year. The monthly payment for the first year is P. The effective annual interest rate is 9%. Calculate P. [4.j #18]

A) 873 B) 803 C) 821 D) 838 E) 856



$P\,s_{\overline{12}|j}$ (using fusion)

$i = 9\%$ $\quad j = 0.7207\%$ $\quad k = 3\%$ $\quad i' = \dfrac{0.09 - 0.03}{1.03} = 5.825\%$

$140{,}000 = \dfrac{P\,s_{\overline{12}|j}}{1.03}\, a_{\overline{26}|i'}$

$P = \boxed{872.99}$