# HW 2.7 (b) Key

1. Mike buys a perpetuity-immediate with varying annual payments. During the first 10 years, the payment is constant and equal to 14. Beginning in year 11, the payments start to increase. For year 11 and all future years, the current year's payment is $K\%$ larger than the previous year's payment. At an annual effect interest rate of 6.1%, the perpetuity has a present value of 500. Calculate $K$, given $K < 6.1$. [4.j #02]

$\boxed{\text{A)}}$ 4.1 B) 4.3 C) 4.5 D) 4.7 E) 4.9

$i = 0.061$

| 0 | 1 | 2 | 3 | ... | 10 | 11 | 12 | 13 | ... |
|---|---|---|---|---|---|---|---|---|---|
| | 14 | 14 | 14 | | 14 | $14(1+k)$ | $14(1+k)^2$ | $14(1+k)^3$ | ... |

$$i' = \frac{0.061 - k}{1+k}$$

$$500 = 14\, a_{\overline{10|}i} + \frac{14\cancel{(1+k)}}{\cancel{1+k}}\, a_{\overline{\infty|}i'} \cdot v^{10}$$

$$i' = 0.0194848$$

$$i' + i'k = 0.061 - k \quad \rightarrow \quad k = \frac{0.061 - i'}{1 + i'} = \boxed{0.04072}$$

2. A senior executive is offered a buyout package by his company that will pay him a monthly benefit for the next 22 years. Monthly benefits will remain constant within each of the 22 years. At the end of each 12-month period, the monthly benefits will be adjusted upwards to reflect the percentage increase in the CPI. You are given:
(i) The first monthly benefit is $R$ and will be paid one month from today.
(ii) The CPI increases 3.1% per year forever.
At an annual effective interest rate of 9%, the buyout package has a value of 102,000. Calculate $R$. [4.j #03]

$\boxed{\text{A)}}$ 682.6 B) 580.2 C) 614.3 D) 648.5 E) 716.7

$i = 9\%$ $k = 3.1\%$ $i' = \dfrac{0.09 - 0.031}{1.031} = 5.7226\%$ $j = 0.7207\%$ (monthly)

AV of pmts in 1st year : $R\, s_{\overline{12|}j} = 12.4873\, R$

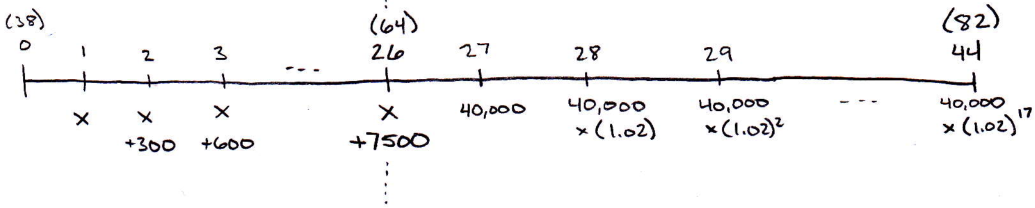
Using Fusion:

$$102{,}000 = \frac{12.4873R}{1.031}\, a_{\overline{22|}i'}$$

$$R = \boxed{682.59}$$

3. Mrs. McNamara just turned 38 and is beginning to plan for her retirement. She would like to make annual contributions to a retirement fund beginning with $X$ on the day she turns 39 and increasing by 300 each year until her last contribution on her 64th birthday. These contributions should fund annual retirement checks beginning with 40,000 on her 65th birthday, and increasing by 2% each year until her last retirement check issued on the day she turns 82. The fund will earn interest at the nominal rate of 7% convertible semiannually. Determine which of the following is closest to the minimum $X$ needed to ensure that Mrs. McNamara's retirement goals are met. [4.j #07]

A) At least 3,600, but less than 4,000
B) At least 2,800, but less than 3,200
C) At least 3,200, but less than 3,600
D) At least 4,000, but less than 4,400
E) At least 4,400, but less than 4,800




$i^{(2)} = 7\%$ $\quad j = 3.5\%$ $\quad i = 7.1225\%$ $\quad k = 2\%$ $\quad i' = 5.02205\%$

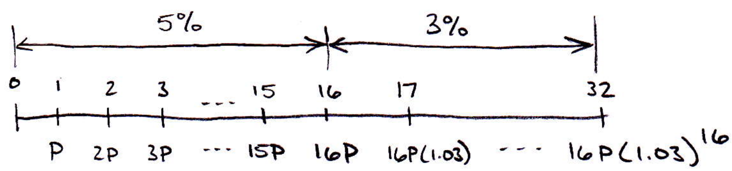
$$X\, s_{\overline{26}|i} + 300\,\frac{s_{\overline{26}|i} - 26}{i} = \frac{40{,}000}{1.02}\, a_{\overline{18}|i'}$$

$$X(69.95736) + 185{,}148.60 = 457{,}626.167$$

$$X = \boxed{3894.91}$$

4. Mary purchases an increasing annuity-immediate for 35,000 that makes 32 annual payments as follows:
(i) $P, 2P, 3P, \ldots, 16P$ in years 1 through 16; and
(ii) $16P(1.03), 16P(1.03)^2, \ldots, 16P(1.03)^{16}$ in years 17 to 32.
The annual effective interest rate is 5% for the first 16 years, and 3% thereafter. Calculate $P$. [4.j #09]

A) 177 B) 166 C) 169 D) 173 E) 180




$k = 0.03$

$$i' = \frac{0.03 - 0.03}{1.03} = 0$$

$$35{,}000 = P(Ia)_{\overline{16}|5\%} + \frac{16P\cancel{(1.03)}}{\cancel{1.03}}\, a_{\overline{16}|0\%} \cdot (1.05)^{-16}$$

$$35{,}000 = P\,\frac{\ddot{a}_{\overline{16}|5\%} - 16v^{16}}{0.05} + (16)^2 P \cdot (1.05)^{-16}$$

$$P = \boxed{176.52}$$

5. Stan elects to receive his retirement benefit over 22 years at the rate of 1900 per month beginning one month from now. The monthly benefit increases by 4% each year. At a nominal interest rate of 7.8% convertible monthly, calculate the present value of the retirement benefit. [4.j #11]

$\boxed{\text{A)}}$ 330,650   B) 291,000   C) 310,800   D) 350,500   E) 370,350

$i^{(12)} = 7.8\%$   $i = 8.0850\%$   $k = 4\%$   $i' = 3.9279\%$

$j = 0.65\%$ (monthly)

AV of pmts in 1st year : $1900\, s_{\overline{12}|j} = 23{,}633.02149$

Using Fusion:

$$PV = \frac{23{,}633.02149}{1.04}\, a_{\overline{22}|i'}$$

$$= \boxed{330{,}662.79}$$