HW 2.8 Key

1. Payments are made to an account at a continuous rate of $(7k + tk)$, where $0 \le t \le 9$. Interest is credited at a force of interest $\delta_t = \dfrac{1}{7+t}$. After 9 years, the account is worth 21,000. Calculate k.

[4.k #01]

$\boxed{\text{A)}}$ 146 B) 126 C) 133 D) 139 E) 152

$$\delta_t = (7+t)^{-1} \rightarrow a(t) = \tfrac{1}{7}(7+t)$$

$$PV = \frac{21{,}000}{a(9)} = 9187.5$$

$$9187.5 = \int_0^9 \frac{7k+tk}{\frac{1}{7}(7+t)}\,dt = \int_0^9 7k\,dt = 63k$$

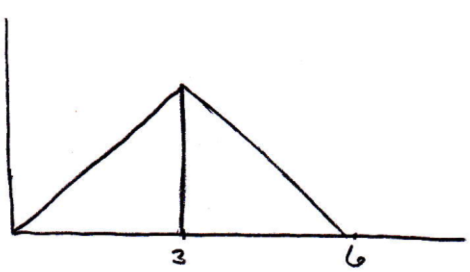
$$k = \boxed{145.83}$$

2. A company is introducing a new product that they think will have a 6-year life cycle, with sales increasing steadily for 3 years, after which they will decline steadily. The company feels that the product will be so successful that they will make sales every day of the year. As a result, they model future sales by assuming net cashflows are received continuously over the 6-year horizon at the following rates:

$140t$ for $0 \le t \le 3$, and $140(6 - t)$ for $3 \le t \le 6$

The company requires an effective annual rate of return on any investment of 7.25%. What is the maximum amount of money the company should spend today to invest in this new product? [4.k #02]

$\boxed{\text{A)}}$ 1,025 B) 1,138 C) 1,251 D) 1,363 E) 1,476



$i = 7.25\%$

$\delta = 6.9992\%$

$$PV = 140(\bar{I}\bar{a})_{\overline{3}|} + 140(\bar{D}\bar{a})_{\overline{3}|} \cdot v^3$$

$$= 140\,\frac{\bar{a}_{\overline{3}|} - 3v^3}{\delta} + 140\,\frac{3 - \bar{a}_{\overline{3}|}}{\delta} \cdot v^3$$

$$= \boxed{1025.12}$$

3. . You are given:

(i) The force of interest at time t is $0.01t$.

(ii) R is the present value of a 6-year continuously increasing annuity which has a rate of payment of $6t$ at time t.

Calculate R. [4.k #03]

$\boxed{\text{A)}}$ 98.84 B) 95.87 C) 101.8 D) 104.77 E) 107.73

$$\delta_t = 0.01t \qquad \int_0^t 0.01r\,dr = 0.005t^2 \qquad a(t) = e^{0.005t^2}$$

$$PV = \int_0^6 6t\,e^{-0.005t^2}\,dt \qquad u = -0.005t^2 \quad du = -0.01t\,dt$$

$$= -\int_0^{-0.18} 600e^u\,du = 600\left[1 - e^{-0.18}\right]$$

$$= \boxed{98.84}$$

4. You are given:

(i) $\bar{a}_{\overline{12|}} = 6.988$; and

(ii) $\frac{d}{d\delta}\left(\bar{a}_{\overline{12|}}\right) = -33.737$.

Calculate δ. [4.d-g #02]

$\boxed{\text{A)}}$ 0.1 B) 0.097 C) 0.098 D) 0.099 E) 0.101

$$\frac{1-v^{12}}{\delta} = 6.988 \rightarrow 1 - e^{-12\delta} = 6.988\,\delta$$

$$\rightarrow e^{-12\delta} = 1 - 6.988\delta$$

$$\frac{d}{d\delta}\left[\frac{1-e^{-12\delta}}{\delta}\right] = \frac{\delta(12e^{-12\delta}) - (1-e^{-12\delta})}{\delta^2}$$

$$= \frac{12\delta(1-6.988\delta) - 1 + (1-6.988\delta)}{\delta^2} = -33.737$$

$$12\delta - 83.856\delta^2 - 6.988\delta = -33.737\delta^2$$

$$50.119\delta^2 - 5.012\delta = 0 \rightarrow \delta = 0 \text{ or } \boxed{\delta = 0.1}$$

(Or you could just plug the answers into $\bar{a}_{\overline{12|}}$. The "table" function in the TI-30XS is VERY useful for this, btw.)

5. You invest in the Esmerelda Corporation by loaning them a certain amount of money, L. They agree to pay you interest each year of 60, payable continuously. Esmerelda Corporation must extinguish the debt by making a single payment of 1300 at the end of either the either year 11, year 12, or year 13. Interest payments stop as soon as the amount of 1300 is paid. You desire at least a 6% rate of return on your investment. Determine the largest amount L can be. [4.d-g #01]

$\boxed{\text{A)}}$ 1156.43 B) 1158.04 C) 1172.09 D) 1164.04 E) 1140.65

$$L_{11} = 60\,\bar{a}_{\overline{11}|} + 1300v^{11} = 1172.09$$

$$L_{12} = 60\,\bar{a}_{\overline{12}|} + 1300v^{12} = 1164.04$$

$$L_{13} = 60\,\bar{a}_{\overline{13}|} + 1300v^{13} = \boxed{1156.43}$$

The "table" function is very useful for calculating these.