HW 2.9(a) Key

1. At an effective interest rate of i, $i > 0$, both of the following annuities have a present value of X:

   (i) a 20-year annuity immediate with annual payments of 40
   (ii) a 30-year annuity immediate with annual payments of 25 per year for the first 10 years, 50 per year for the second 10 years, and 75 per year for the final 10 years.

   Calculate X. [3.g #01]

   $\boxed{\text{A)}}$ 341 B) 320 C) 327 D) 334 E) 347

$$X = 40a_{\overline{20}|i}$$

$$X = 25a_{\overline{10}|i} + 50a_{\overline{10}|i}v^{10} + 75a_{\overline{10}|i}v^{20}$$

$$40a_{\overline{20}|} = 25a_{\overline{10}|} + 50a_{\overline{10}|}v^{10} + 75a_{\overline{10}|}v^{20}$$

$$40(1+v^{10}) = 25 + 50v^{10} + 75v^{20}$$

$$75v^{20} + 10v^{10} - 15 = 0$$

$$v^{10} = 0.38549 \rightarrow i = 10\% \rightarrow X = 40a_{\overline{20}|10\%} = \boxed{340.54}$$

2. You are given:
   (i) $a_{\overline{n}|} = 20.00$; and
   (ii) $a_{\overline{3n}|} = 48.80$.

   Determine $a_{\overline{4n}|}$. [3.g #04]

   $\boxed{\text{A)}}$ 59.04 B) 57.27 C) 57.86 D) 58.45 E) 59.63

$$\frac{a_{\overline{3n}|}}{a_{\overline{n}|}} = 1 + v^n + v^{2n} = 2.44$$

$$v^{2n} + v^n - 1.44 = 0 \rightarrow v^n = 0.8$$

$$\frac{a_{\overline{4n}|}}{a_{\overline{n}|}} = 1 + v^n + v^{2n} + v^{3n} = 2.952$$

$$a_{\overline{4n}|} = 2.952(a_{\overline{n}|}) = 2.952(20) = \boxed{59.04}$$

3. At an effective annual interest rate i, you are given:
   (i) the present value of an annuity-immediate with annual payments of 1 for n years is 50.00; and
   (ii) the present value of an annuity-immediate with annual payments of 1 for $3n$ years is 68.37.
   Calculate the accumulated value of an annuity-immediate with annual payments of 1 for $2n$ years. [3.g #06]

$\boxed{\text{A)}}$ 787   B) 764   C) 772   D) 780   E) 795

$50 = a_{\overline{n}|}$

$68.37 = a_{\overline{3n}|}$

$\frac{a_{\overline{3n}|}}{a_{\overline{n}|}} = 1 + v^n + v^{2n} = 1.3674$

$v^{2n} + v^n - 0.3674 = 0 \rightarrow v^n = 0.2857$

$50 = \frac{1-v^n}{i} = \frac{1-0.2857}{i}$

$i = 0.014285$

$v^n = 0.2857 \rightarrow (1+i)^n = 3.5$

$s_{\overline{2n}|} = \frac{(3.5)^2 - 1}{i} = \boxed{787.54}$

4. Samantha receives a 18,000 life insurance benefit. If she uses the proceeds to buy an n-year immediate annuity, the annual payout will be 2024. If a $2n$-year immediate annuity is purchased, the annual payout will be 1522. Both calculations are based on an effective annual interest rate of i.

   Calculate i. [3.g #07]

$\boxed{\text{A)}}$ 0.075   B) 0.074   C) 0.077   D) 0.078   E) 0.08

$18000 = 2024\, a_{\overline{n}|}$

$18,000 = 1522\, a_{\overline{2n}|}$

$1522\, a_{\overline{2n}|} = 2024\, a_{\overline{n}|}$

$\frac{a_{\overline{2n}|}}{a_{\overline{n}|}} = 1 + v^n = 1.32983$

$v^n = 0.32983$

$18,000 = 2024 \frac{1-v^n}{i}$

$\boxed{i = 0.07536}$

5. At a rate of interest, i, where $i > 0$, a 44-year annuity immediate with annual payments of \$9 has the same present value as a 22-year annuity-immediate with annual payments of \$11.

   In how many year does money double at rate of interest, i? [3.g #08]

$\boxed{\text{A)}}$ 10.14   B) 8.11   C) 8.62   D) 9.12   E) 9.63

$9\, a_{\overline{44}|} = 11\, a_{\overline{22}|}$

$\frac{a_{\overline{44}|}}{a_{\overline{22}|}} = 1.22222$

$1 + v^{22} = 1.22222$

$i = 7.0758\%$

$(1.070758)^t = 2$

$\boxed{t = 10.1386}$