HW 2.9 (b) Key

1. Given the information below, find $\bar{s}_{\bar{1}|}$.

$$\ddot{a}_{\overline{n+3|}} = 12.4379 \qquad \ddot{s}_{\overline{n+1|}} = 45.6664$$

[4.d-g #05]

$$\ddot{a}_{\overline{n+3|}} = 12.4379 \quad \rightarrow \quad a_{\overline{n+2|}} = 11.4379$$

$$\ddot{s}_{\overline{n+1|}} = 45.6664 \quad \rightarrow \quad s_{\overline{n+2|}} = 46.6664$$

$$\frac{s_{\overline{n+2|}}}{a_{\overline{n+2|}}} = (1+i)^{n+2} = 4.07998$$

$$s_{\overline{n+2|}} = \frac{(1+i)^{n+2} - 1}{i} = \frac{3.07998}{i} = 46.6664 \rightarrow i = 6.6\%$$

$$\bar{s}_{\overline{1}|} = \frac{i}{\delta} = \frac{i}{\ln(1+i)} = \boxed{1.0326}$$

2. You are given, for interest rate i:

   (i) $\ddot{a}_{\overline{n|}} = 8.6993$

   (ii) $\ddot{a}_{\overline{2n|}} = 11.0228$

   Calculate i. [4.d-g #07]

   A) 9.2%   B) 8.92%   C) 9.02%   D) 9.11%   E) 9.29%

$$\frac{\ddot{a}_{\overline{2n|}}}{\ddot{a}_{\overline{n|}}} = \frac{a_{\overline{2n|}}}{a_{\overline{n|}}} = 1 + v^n = 1.26709 \rightarrow v^n = 0.26709$$

$$\ddot{a}_{\overline{n|}} = \frac{1 - v^n}{d} = \frac{0.7329}{d} = 8.6993$$

$$d = 0.08425$$

$$i = \boxed{0.092}$$

3. Peter borrows $2600 from Kevin for a term of 8 years. Peter agrees to pay interest at the end of each year at an effective annual interest rate of 10% and to repay the entire $2600 as a lump sum at the end of 8 years. Immediately after payment number 4, Kevin sells his rights to future payments to Martha at a price that will yield Martha an effective annual rate of return of 4%. Determine Kevin's overall yield rate. [3.h-i #01]

A) At least 14%, but less than 14.5%   D) At least 15.5%, but less than 16%
B) At least 14.5%, but less than 15%   E) At least 16%, but less than 16.5%
C) At least 15%, but less than 15.5%

Original Terms:

Martha:

$$X = 2600a_{\overline{4}|4\%} + 2600v^4$$
$$X = 3166.2637 \quad (\text{using BA II})$$

Kevin:

$$2600 = 2600a_{\overline{4}|}i + Xv^4$$
$$\boxed{i = 14.4\%} \quad (\text{using BA II})$$

4. On 1/1/08, Cathy borrows $1400 from ABS Finance Company and agrees to pay it back in 10 annual installments at an annual effective rate of 10%. The first payment is due one year after the loan is issued. On 1/1/11 Cathy pays off the balance of the loan plus a $55 prepayment penalty. Find ABC's effective overall yield rate i. [3.h-i #04]

A) At least 11%, but less than 11.5%   D) At least 12.5%, but less than 13%
B) At least 11.5%, but less than 12%   E) At least 13%, but less than 13.5%
C) At least 12%, but less than 12.5%

Original Terms: $1400 = Pa_{\overline{10}|10\%} \rightarrow P = 227.8436$

New Terms:

(B_3 is the outstanding balance of the loan at $t=3$)

Note that B_3 would be paid off by 7 more payments of P.

So, B_3 is the PV of the last 7 payments: $B_3 = Pa_{\overline{7}|10\%} = 1109.2378$

$$1400 = Pa_{\overline{3}|}i + (1109.2378 + 55)v^3$$
$$1400 = 227.8436 a_{\overline{3}|}i + 1164.2378 v^3 \rightarrow \boxed{i = 11.2478}$$
$$(\text{using BA II})$$

5. A 10,000 loan is to be repaid by 15 equal payments at the end of each year. The outstanding balance is amortized at 8.5%.

In addition to the annual payments, the borrower must pay an origination fee at the time the loan is made. The fee is 2.42% of the loan but does not reduce the loan balance.

Immediately after payment number 8, the borrower pays the remaining loan balance.

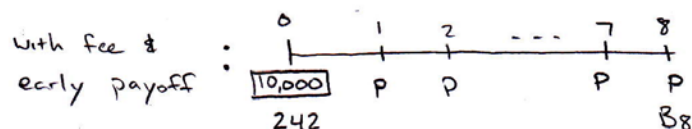Determine the yield to the lender considering the origination fee and the early pay-off of the loan. [3.h-i #09]

A) 9%   B) 8.9%   C) 9.1%   D) 9.2%   E) 9.3%

Original Terms (w/o fee): $10,000 = P\, a_{\overline{15}|\,8.5\%}$ → $P = 1204.2046$

Fee: $10,000(0.0242) = 242$ (PV of last 7 pmts)

Remaining Balance at $t=8$: $B_8 = P\, a_{\overline{7}|\,8.5\%} = 6163.7376$

with fee &
early payoff

$10,000 = 242 + P\, a_{\overline{8}|\,i} + B_8\, v^8$

$9758 = P\, a_{\overline{8}|\,i} + B_8\, v^8$

$\boxed{i = 9\%}$ (BA II)