HW 2.9 (c) Key

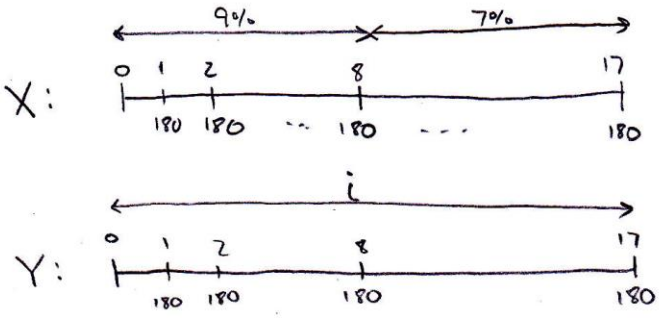
1. Jeff deposits 180 at the end of each year for 17 years into Fund X. Antoinette deposits 180 at the end of each year for 17 years into Fund Y.

Fund X earns an annual effective rate of 9% for the first 8 years and an annual effective rate of 7% thereafter. Fund Y earns an annual effective rate of i.

At the end of 17 years, the accumulate value of Fund X equals the accumulated value of Fund Y.

Calculate i. [3.j #02]

A) 7.5% B) 7.3% C) 7.7% D) 7.9% E) 8.2%



$180\, s_{\overline{8}|9\%}(1.07)^9 + 180\, s_{\overline{9}|7\%} = 180\, s_{\overline{17}|i}$

$\boxed{i = 7.4983\%}$ (BA II)

2. You are given $\delta_t = \dfrac{2}{7+t}, t \geq 0$.

Calculate $a_{\overline{3}|}$. [3.j #05]

A) 1.86 B) 1.95 C) 2.05 D) 2.14 E) 2.23

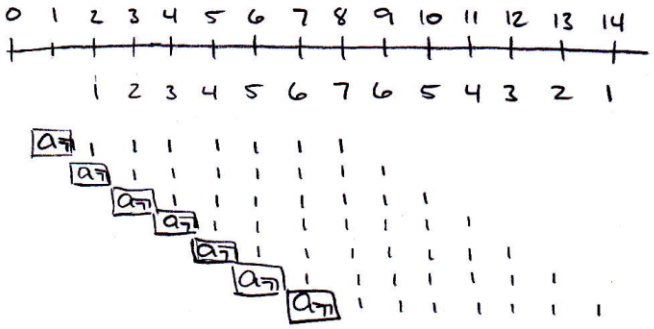
$\int_0^r \delta_r\, dr = 2\ln(7+r)\Big|_0^t = 2\ln(7+t) - 2\ln(7)$

$a(t) = e^{2\ln(7+t) - 2\ln(7)} = \frac{1}{49}(7+t)^2$

$a_{\overline{3}|} = \frac{1}{a(1)} + \frac{1}{a(2)} + \frac{1}{a(3)} = \frac{49}{64} + \frac{49}{81} + \frac{49}{100} = \boxed{1.8606}$

3. A 13 year annuity has a series of payments 1, 2, 3, ... , 6, 7, 6, ... , 3, 2, 1, with the first payment made at the end of the second year. The present value of this annuity is 28.78 at interest rate *i*. A 14-year annuity has a series of payments 1, 2, 3, ... , 6, 7, 7, 6, ... , 3, 2, 1, with the first payment made at the end of the first year. Calculate the present value of the 14-year annuity at interest rate *i*. [4.1 #01]

A) 34.1 B) 32.8 C) 33.5 D) 34.8 E) 35.5

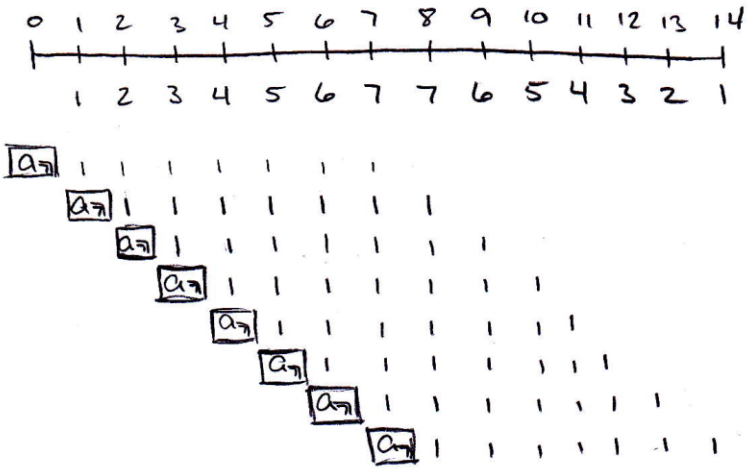


$28.78 = (a_{\overline{7}|})^2$

$a_{\overline{7}|} = 5.3647$

$i = 7.1313\%$

↑ we don't actually need this.



$PV = a_{\overline{7}|}\ddot{a}_{\overline{8}|}$

$= a_{\overline{7}|}(a_{\overline{7}|} + 1)$

$= \boxed{34.14}$

4. A fund is built with annual payments increasing by 1 from 1 to 11, and then decreasing by 1 to 0. The first payment of 1 is made today. If the fund is used to purchase a 11 year level annuity with the first payment 27 years from today, what is the amount of the level payment? Assume an annual effective rate of interest of 6%. [4.1 #02]

A) At least 40, but less than 41
B) At least 38, but less than 39
C) At least 39, but less than 40
D) At least 41, but less than 42
E) At least 42, but less than 43

$(\ddot{a}_{\overline{11}|})^2 = X\, a_{\overline{11}|6} \cdot v^{26}$

$X = \boxed{40.315}$

5. You are given $\delta_t = 1/(5+t)$ for $0 \le t \le 4$. Calculate $s_{\overline{4}|}$. [4.d-g #03]

A) 4.9 B) 5.2 C) 5.4 D) 5.6 E) 5.9

$$\int_0^t \delta_r \, dr = \ln(5+r)\Big|_0^t = \ln(5+t) - \ln(5)$$

$$a(t) = \frac{1}{5}(5+t)$$

$$a_{\overline{4}|} = \frac{1}{a(1)} + \frac{1}{a(2)} + \frac{1}{a(3)} + \frac{1}{a(4)}$$

$$= \frac{5}{6} + \frac{5}{7} + \frac{5}{8} + \frac{5}{9} = 2.72817$$

$$s_{\overline{4}|} = a_{\overline{4}|}(a(4)) = 2.72817 \, \frac{9}{5} = \boxed{4.91}$$