HW 3.2 (d) Key

1. Dave takes out a 25-year, $70,000 mortgage with monthly payments beginning one month from now. He makes regular payments until and including the first payment in which the principal portion exceeds the interest portion of the payment. You may assume $i^{(12)} = 0.051$. $j = 0.00425$

   Determine the total interest Dave has paid on the mortgage after making the first payment in which the principal portion exceeds the interest portion of the payment. [6.a #17]

   $\boxed{\text{A)}}$ At least $35,250, but less than $35,500
   B) At least $35,000, but less than $35,250
   C) At least $35,500, but less than $35,750
   D) At least $35,750, but less than $36,000
   E) At least $36,000, but less than $36,250

$70{,}000 = R\,a_{\overline{300}|j} \rightarrow R = 413.30$

Find t such that $P_t > I_t$:

$R v^{301-t} = R(1 - v^{301-t}) \rightarrow 2v^{301-t} = 1 \rightarrow v^{301-t} = \frac{1}{2}$

$\rightarrow (1+j)^{301-t} = 2 \rightarrow t = 137.56 \rightarrow$ use $t = 138$

$B_{138} = R\,a_{\overline{162}|j} = 48{,}325.72$

Total Interest $= 138R + 48{,}325.72 - 70{,}000 = \boxed{35{,}361.12}$

2. Donald takes out a loan to be repaid with annual payments of 1000 at the end of each year for $2n$ years. The annual effective interest rate is 5.9%. The sum of the interest paid in year 1 plus the interest paid in year $n + 1$ is equal to 1610. Calculate the amount of interest paid in year 8. [6.a #29]

   $\boxed{\text{A)}}$ 866 B) 917 C) 969 D) 1,021 E) 1,073

$R = 1000$ $i = 5.9\%$

$I_1 + I_{n+1} = 1610 \rightarrow 1000(1 - v^{2n+1-1}) + 1000(1 - v^{2n+1-(n+1)}) = 1610$

$\rightarrow 1 - v^{2n} + 1 - v^n = 1.61 \rightarrow v^{2n} + v^n - 0.39 = 0 \rightarrow v^n = 0.3$

$v^n = 0.3 \rightarrow n = 21$

$I_8 = 1000(1 - v^{35}) = \boxed{865.53}$

Remember, loan term
is $\underline{2n}$ years.

3. Sam borrowed \$1400 on January 1, 2013 to be repaid by level payments every two years beginning January 1, 2015, at an effective annual rate of interest of 7%. The amount of interest in payment number 5 is \$196.97. Determine the amount of principal in installment number 10. [6.a #30]

$\boxed{\text{A)}}$ At least \$25, but less than \$30
B) At least \$20, but less than \$25
C) At least \$30, but less than \$35
D) At least \$35, but less than \$40
E) At least \$40, but less than \$45

$i = 7\%$ $\quad j = 14.49\%$

$1400 = R\,a_{\overline{n}|j} \rightarrow 202.86 = R(1-v^n)$

$196.97 = I_5 \rightarrow 196.97 = R(1-v^{n-4})$

Divide $\rightarrow 1.0299 = \dfrac{1-v^n}{1-v^n v^{-4}}$

$\rightarrow 1.0299(1-1.71819v^n) = 1-v^n \rightarrow v^n = 0.03886 \rightarrow n = 24 \rightarrow R = 211.06$

$P_{10} = 211.06\,v^{15} = \boxed{27.73}$

4. A loan is to be repaid by annual installments of X at the end of each year for 18 years. You are given:
   (i) The total principal repaid in the first 3 years is 190.23; and
   (ii) The total principal repaid in the last 3 years is 524.86.
   Calculate the total amount of interest paid during the life of the loan. [6.a #46]

$\boxed{\text{A)}}$ 1,588 B) 1,676 C) 1,763 D) 1,850 E) 1,938

$P_1 + P_2 + P_3 = 190.23 \rightarrow X[v^{18} + v^{17} + v^{16}] = 190.23$

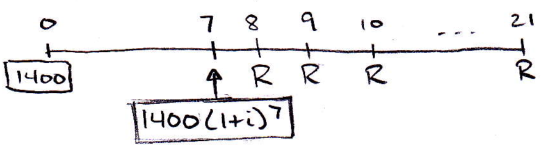
$P_{16} + P_{17} + P_{18} = 524.86 \rightarrow X[v^3 + v^2 + v^1] = 524.86$

$v^{15} = 0.3624 \rightarrow i = 7\%$

$X = 200 \rightarrow L = 200\,a_{\overline{18}|7\%} = 2011.82 \quad \Sigma I_t = 200(18) - L = \boxed{1588.18}$

5. Bob takes out a loan of 1,400 at an annual effective interest rate of i. You are given:
   (i) The first payment is made at the end of year 8;
   (ii) 14 equal annual payments are made to repay the loan in full at the end of year 21; and
   (iii) The outstanding principal after the payment made at the end of year 14 is 1178.97.
   Calculate the outstanding principal at the end of year 7. [6.a #47]

$\boxed{\text{A)}}$ 2,005 B) 1,910 C) 1,955 D) 2,050 E) 2,095



$1400(1+i)^7 = R\,a_{\overline{14}|}$

$1178.97 = R\,a_{\overline{7}|}$

$\dfrac{R\,a_{\overline{14}|}}{R\,a_{\overline{7}|}} = \dfrac{1400(1+i)^7}{1178.97} \rightarrow 1178.97(1+v^7) = 1400(1+i)^7 \xrightarrow{\times v^7}$

$\rightarrow 1178.97v^7 + 1178.97v^{14} = 1400 \rightarrow v^7 = 0.6989 \rightarrow v = 0.95012$

$\rightarrow i = 5.25\% \rightarrow 1400(1+i)^7 = \boxed{2003}$