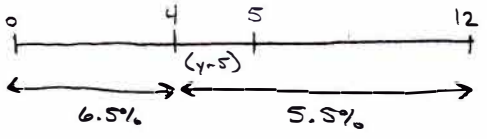
HW 3.3 (a) Key

1. A 12-year loan of 24,000 is to be repaid with payments at the end of each year consisting of interest on the loan and a sinking fund deposit. Interest on the loan is charged at a 8.5% annual effective rate. The sinking fund's annual effective interest rate is 6.5%. However, beginning in year 5, the annual effective interest rate on the sinking fund drops to 5.5%. As a result, the annual payment to the sinking fund is then increased by X. Calculate X. [6.b #04]

A) 126 B) 82 C) 93 D) 104 E) 115

$24{,}000 = SFD\ s_{\overline{12}|6.5\%} \rightarrow SFD = 1381.64$



$$24{,}000 = 1381.64\, s_{\overline{4}|6.5\%}(1.055)^8 + (X + 1381.64)\, s_{\overline{8}|5.5\%}$$

$$X = \boxed{125.84}$$

2. Jason and Margaret each take out a 20-year loan of L. Jason repays his loan using the amortization method, at an annual effective interest rate of i. He makes an annual payment of 350 at the end of each year. Margaret repays her loan using the sinking fund method. She pays interest annually, also at an annual effective interest rate of i. In addition, Margaret makes level annual deposits at the end of each year for 20 years into a sinking fund. The annual effective rate on the sinking fund is 4%, and she pays off the loan after 20 years. Margaret's total payment each year is equal to 10.5% of the original loan amount. Calculate L. [6.b #05]

A) 3,665 B) 3,500 C) 3,585 D) 3,750 E) 3,835

Jason: $L = 350\, a_{\overline{20}|i}$

Margaret: $I = Li \qquad L = SFD\ s_{\overline{20}|4\%}$

$$I + SFD = 0.105L$$

$$Li + \frac{L}{s_{\overline{20}|4\%}} = 0.105L$$

$$i + \frac{1}{s_{\overline{20}|4\%}} = 0.105 \rightarrow i = 7.1418\%$$

$$L = 350\, a_{\overline{20}|i} = \boxed{3667.38}$$

3. On January 1, 2013, Barbara takes out a $12,000 loan, at an annual effective interest rate of 6.5%. Interest is being paid annually beginning on January 1, 2014. In addition, a sinking fund is established to repay the principal on January 1, 2023. Payments are made into the sinking fund annually, with the first payment on January 1, 2014, and the last payment on January 1, 2023. The sinking fund earns an effective annual interest rate of 4.5%. From Barbara's point of view, what rate of interest is she really paying each year? [6.b #07]

A) At least 7.5%, but less than 8%
B) At least 8%, but less than 8.5%
C) At least 8.5%, but less than 9%
D) At least 9%, but less than 9.5%
E) At least 9.5%, but less than 10%

$n = 10 \quad L = 12{,}000 \quad i = 0.065$

$I = Li = 780 \qquad 12{,}000 = SFD\, s_{\overline{10}|4.5\%} \rightarrow SFD = 976.55$

$I + SFD = 1756.55$

$12{,}000 = 1756.55\, a_{\overline{10}|i}$

$i = \boxed{7.6041\%}$

4. John borrows 20,000 for 18 years at an annual effective interest rate of i. He accumulates the amount necessary to repay the loan by using a sinking fund. He makes 18 payments of X at the end of each year, which includes interest on the loan and the payment into the sinking fund, which earns an annual effective rate of 8%. If the annual effective rate of the loan had been $2i$, his total annual payment would have been $1.7X$. Calculate i. [6.b #09]

A) 6.2% B) 6.4% C) 6.6% D) 6.8% E) 7%

$20{,}000 = SFD\, s_{\overline{18}|8\%} \rightarrow SFD = 534.04$

$\times 1.7 \quad (X = 534.04 + 20{,}000\, i)$

$1.7X = 534.04 + 40{,}000\, i$

$\downarrow$

$0 = 373.828 - 6000\, i$

$i = \boxed{6.23\%}$

5. Helen borrows \$18,000 to be repaid over 16 years with level annual payments with an annual effective interest rate of 7%. The first payment is due one year after she takes out the loan. Helen pays an additional \$6,000 at the end of year 5 (in addition to her normal payment). At that time (the end of year 5) she negotiates to pay off the remaining principal at the end of year 12 with a sinking fund. The sinking fund accumulates at an annual effective interest rate of 6%. Helen will make level annual payments. Helen will also make annual interest payments at an annual effective interest rate of 9%. You may assume all payments are made at the end of the year. Determine Helen's total annual outlay starting with year 6. [6.b #11]

A) At least \$1,725, but less than \$1,750
B) At least \$1,650, but less than \$1,675
C) At least \$1,675, but less than \$1,700
D) At least \$1,700, but less than \$1,725
E) At least \$1,750, but less than \$1,775

Original Terms: $18{,}000 = R\,a_{\overline{16}|7\%} \rightarrow R = 1905.44$

$B_5 = R\,a_{\overline{11}|7\%} = 14{,}288.26$

Amount owed after one-time payment: 8288.26

SF Method: $I = 8288.26(0.09) = 745.94$

$8288.26 = SFD\ s_{\overline{7}|6\%} \rightarrow SFD = 987.42$

$745.94 + 987.42 = \boxed{1733.36}$