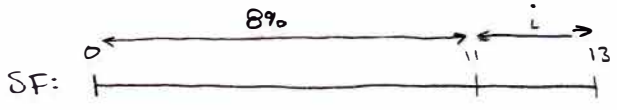
HW 3.3 (b) Key

(This space for rent.)

1. Ron takes out a 13-year loan for 7000. Interest on the loan is charged at a 9% annual effective rate. Ron repays the loan by making 13 annual payments at the end of each year consisting of interest on the loan and a sinking fund deposit. The total payment each year is 973. The amount in the sinking fund will exactly pay off the loan at the end of 13 years. During the first 11 years of the loan, the sinking fund annual effective interest rate is 8%. After year 11, the sinking fund annual effective rate is i. Calculate i, $i > 0$. [6.b #13]

A) 5.02% B) 4.85% C) 4.93% D) 5.1% E) 5.19%



$I = 7000(0.09) = 630$ $\quad SFD = 973 - 630 = 343$

$7000 = 343\, s_{\overline{11}|8\%}(1+i)^2 + 343\, s_{\overline{2}|i}$

$7000 = 5709.40(i^2+2i+1) + 343(2+i)$

$5709.40\, i^2 + 11{,}761.80\, i - 604.6 = 0$

$i = \boxed{5.018\%}$

2. Judy buys an $9,000 car on a 6-year "lease with option to buy" arrangement which requires her to pay $110 per month to cover the interest (i = 9.9% convertible monthly) plus a portion of the principal. Judy sets up an 9% (convertible monthly) sinking fund with monthly payments P, in order to accumulate the remaining principal at the end of the lease. Assuming the first car payment and sinking fund deposit are made at the end of the first month, find P. [6.b #19]

[A] At least $55, but less than $60
B) At least $40, but less than $45
C) At least $45, but less than $50
D) At least $50, but less than $55
E) At least $60, but less than $65

$i^{(12)} = 9.9\%$ $\quad j = 0.825\%$

Amount owed after 6 years $= 9000(1+j)^{72} - 110\, s_{\overline{72}|j} = 5503.82$

SF: $i^{(12)} = 9\%$ $\quad j = 0.75\%$

$5503.82 = SFD\; s_{\overline{72}|0.75\%} \rightarrow SFD = \boxed{57.93}$

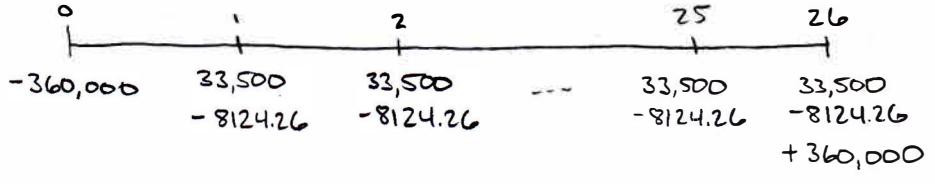
3. An investor pays $360,000 for a 26-year annuity-immediate with annual payments of $33,500. The investor accumulates a sinking fund by making level deposits at the end of each year, so as to replace his capital by the end of the 26-year period. The annual rate of interest on the sinking fund is 4%. Find the yield rate on the investment. [6.b #25]

[A] 7% B) 5.6% C) 6.1% D) 6.6% E) 7.5%

Investor pays 360,000 to receive 33,500/yr for 26 yrs.

Sinking Fund: $360{,}000 = SFD\; s_{\overline{26}|4\%} \rightarrow SFD = 8124.26.$

Transaction Summary:



$360{,}000 = 25{,}375.74\, a_{\overline{26}|i} + 360{,}000\, v^{26}$

$i = \boxed{7.0488\%}$

4. A company agrees to repay a loan over 11 years. Interest payments are made annually and a sinking fund is built with 11 equal annual payments made at the end of each year. Interest on the sinking fund is compounded annually. You are given:
(i) The amount in the sinking fund immediately after the first payment is X.
(ii) The amount in the sinking fund immediately after the second payment is Y.
(iii) $Y/X = 2.05$.
(iv) The net amount of the loan immediately after payment number 8 is 2644.78.
Calculate the amount of the sinking fund payment. [6.d #04]

A) 568 B) 489 C) 516 D) 542 E) 594

$SFD = X \qquad Y = X\, s_{\overline{2}|i} \rightarrow Y/X = 2.05 = s_{\overline{2}|i} \rightarrow i = 5\%$

$NB_8 = L - SFB_8 \qquad L = X\, s_{\overline{11}|5\%}$

$2644.78 = L - X\, s_{\overline{8}|5\%}$

$2644.78 = X\, s_{\overline{11}|5\%} - X\, s_{\overline{8}|5\%}$

$X = \boxed{567.83}$

5. A corporation borrows 15,000 for 23 years, at an effective annual interest rate of 9%. A sinking fund is used to accumulate the principal by means of 23 annual deposits earning an effective annual interest rate of 7%. Calculate the sum of the net amount of interest paid in installment number 15 and the increment in the sinking fund for year 7. [6.d #05]

A) 1,328 B) 1,288 C) 1,308 D) 1,348 E) 1,368

$I = 15{,}000(0.09) = 1350$

$15{,}000 = SFD\, s_{\overline{23}|7\%} \rightarrow SFD = 280.71$

$SFB_{14} = 6330.12 \qquad SFI_{15} = 6330.12(0.07) = 443.11$

$SFB_6 = 2007.99 \qquad NI_{15} = 1350 - 443.11 = 906.89$

$SFB_7 = 2429.26 \qquad Increment = SFB_7 - SFB_6 = 421.27$

$906.89 + 421.27 = \boxed{1328.16}$

These can be found very quickly using BA II.