# HW 3.4 (b) Key

(This space for rent.)

Ellen borrows $12,675 for 13 years at a 6% effective annual rate. She pays interest yearly and will repay the principal by making 13 annual payments into a sinking fund. The first interest and sinking fund payments will be made at the end of the first year. Ellen's sinking fund payments will start at $975 and will decrease by $75 each year. The final payment will include an additional amount. The sinking fund will earn interest at a 4% effective annual rate for the first 7 years and will earn interest at a 5% effective annual rate after the first 7 years. Determine the total amount of Ellen's final sinking fund payment.
[6.e #08]

A) At least $2,800, but less than $2,900   D) At least $2,600, but less than $2,700
B) At least $2,400, but less than $2,500   E) At least $2,700, but less than $2,800
C) At least $2,500, but less than $2,600
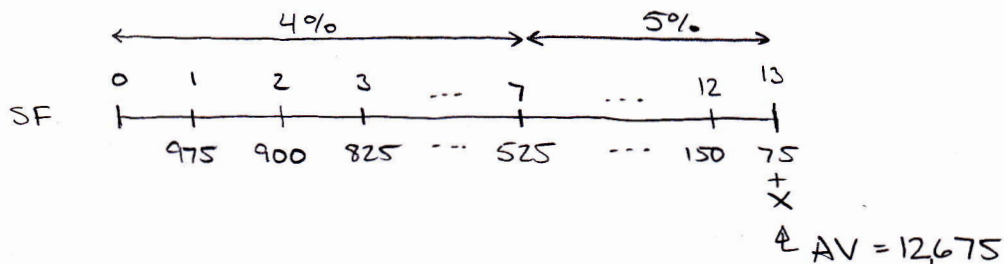
$$\left[975\, s_{\overline{7}|4\%} - 75\, \frac{s_{\overline{7}|4\%} - 7}{0.04} \right](1.05)^6 + 75(Ds)_{\overline{6}|5\%} + X = 12{,}675$$

$$8062.73 + 1857.99 + X = 12{,}675$$

$$X = 2754.28$$

$$X + 75 = \boxed{2829.28}$$

2. A loan of 60,000 is repaid with unequal annual payments at the end of each year for 23 years. Each of the first 22 payments is equal to two times the amount of interest then due. The final payment repays the remaining loan balance at that time. Interest is charged at an annual effective interest rate of 4%. Calculate the amount of the final loan payment. [6.e #10]

A) 25,420   B) 24,910   C) 25,930   D) 26,440   E) 26,940

Note: Each payment reduces balance by 4%.

$$B_t = 60,000 (0.96)^t$$

$$B_{22} = 24,440.97$$

$$R_{23} = B_{22}(1.04) = \boxed{25,418.61}$$

3. A loan of 60,000 is being repaid by a 32-year increasing annuity-immediate. The initial payment is k, and each subsequent payment is k larger than the preceding payment. Determine the principal outstanding immediately after payment number 19, using an annual effective interest rate of 6%. [6.e #13]

A) 80,520   B) 78,910   C) 79,320   D) 79,720   E) 80,120

$$60,000 = k\,(Ia)_{\overline{32}|6\%} \quad \rightarrow \quad k = 361.07$$

$$B_{19} = 20k\,a_{\overline{13}|6\%} + k\,\frac{a_{\overline{13}|6\%} - 13v^{13}}{0.06}$$

$$= \boxed{80,523.93}$$

4. Warren has a loan with an effective interest rate of 10% per annum. He makes payments at the end of each year for 13 years. The first payment is 160, and each subsequent payment increases by 20 per year. Calculate the interest portion in payment number 9. [6.e #16]

A) 135   B) 130   C) 132   D) 138   E) 140
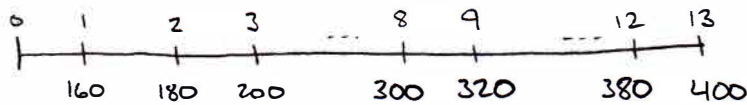
$$L = 160\, a_{\overline{13}|10\%} + 20\, \frac{a_{\overline{13}|10\%} - 13 v^{13}}{0.1} \quad \leftarrow \text{Don't need.}$$

$$B_8 = 320\, a_{\overline{5}|10\%} + 20\, \frac{a_{\overline{5}|10\%} - 5 v^5}{0.1} = 1350.29$$

$$I_9 = i\, B_8 = \boxed{135}$$

5. Lesley borrows 70,000 at an effective annual interest rate of 6%. At the end of each year Lesley pays the interest on the loan. In addition to paying the annual interst amount, Lesley deposits X at the beginning of each year into a sinking fund. The sinking fund pays interest at an effective annual interest rate of 4%. At the end of each year the sinking fund interest is withdrawn and used to reduce the principal on the loan. At the end of 19 years the balance in the sinking fund is sufficient to pay off the balance of the loan. Calculate X. [6.e #19]

A) 2,630   B) 2,160   C) 2,275   D) 2,395   E) 2,515
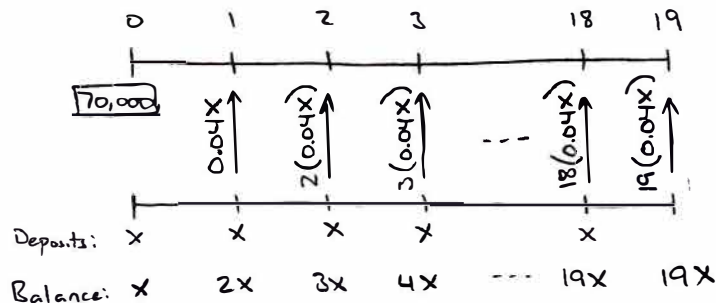
$$70{,}000 - 0.04 X \left(1 + 2 + 3 + \ldots + 18 + 19\right) = 19 X$$

$$70{,}000 - 0.04 X \left(190\right) = 19 X$$

$$X = \boxed{2631.58}$$