HW 3.4 (c) Key

1. A 19-year loan of 6,650 is to be repaid with payments at the end of each year. It can be repaid under the following two options:
   (i) Equal payments at an annual effective rate of 4%.
   (ii) Installments of 350 each year plus interest on the unpaid balance at an annual effective rate of *i*.
   The sum of the payments under option (i) equals the sum of the payments under option (ii). Determine *i*. [6.f #01]

   $\boxed{\text{A)}}$ 4.47% B) 3.98% C) 4.1% D) 4.23% E) 4.35%

i) $6650 = R\, a_{\overline{19|}4\%} \longrightarrow R = 506.32$

ii)

	0	1	2	...	18	19
B_t:	6650	6300	5950	...	350	0
I_t:		$6650i$	$6300i$	...	$700i$	$350i$
P_t:		350	350	...	350	350

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$19(506.32) = 350(19) + 350i(1+2+\dots+18+19)$$

$$19(506.32) = 350(19) + 350(190)i \rightarrow \boxed{i = 4.47\%}$$

2. Two loans for equal amounts are repaid at an effective interest rate of 9.5%. Loan 1 is to be repaid with 34 equal annual payments. Loan 2 is to be repaid by 34 annual payments, each containing equal principal amounts and an interest amount based on the unpaid balance. Payments are made at the end of each year. The annual payment for Loan 1 first exceeds the annual payment for Loan 2 with payment number *n*. Determine *n*. [6.f #03]

   $\boxed{\text{A)}}$ 10 B) 9 C) 11 D) 12 E) 13

Loan 1: $L = R\, a_{\overline{34|}9.5\%} \rightarrow R = \dfrac{L}{10.0453}$

Loan 2: $R_t = \dfrac{L}{34} + \dfrac{35-t}{34} L(0.095)$

$$R > R_t \Rightarrow \frac{L}{10.0453} > \frac{L}{34} + \frac{35-t}{34} L(0.095)$$

$$\frac{34}{10.0453} > 1 + (35-t)(0.095)$$

$$3.3847 > 1 + 3.325 - 0.095t$$

$$0.095t > 0.9403$$

$$t > 9.898 \qquad \boxed{n = 10}$$

3. Frances borrows $10,400 and agrees to make 26 equal annual payments toward principal, where the first payment is due in one year. In addition to the principal repayments, each year she will pay interest at 7% effective on the outstanding principal. The lender wishes to sell the loan to an investor immediately after the loan is made. Determine the sale price such that the investor will achieve a yield of 8% on the investment. [6.f #04]

$\boxed{\text{A)}}$ At least $9,600, but less than $9,700	D) At least $9,500, but less than $9,600
B) At least $9,300, but less than $9,400	E) At least $9,700, but less than $9,800
C) At least $9,400, but less than $9,500	

	0	1	2	3	...	25	26
B_t:	10,400	10,000	9,600	9,200	...	400	0
I_t:		728	700	672	...	52	26
P_t:		400	400	400	...	400	400

$$P = 1128\, a_{\overline{26|}8\%} - 28\,\frac{a_{\overline{26|}8\%} - 26v^{26}}{0.08}$$

$$= \boxed{9640.50}$$

4. A loan of 2800 is being repaid in 14 years by semiannual installments of 100, plus interest on the unpaid balance at 5% per annum compounded semiannually. The installments and interest payments are reinvested at 6% per annum compounded semiannually. Calculate the effective yield rate of the loan. [6.f #05]

$\boxed{\text{A)}}$ 0.057 B) 0.047 C) 0.049 D) 0.052 E) 0.054

$j = 2.5\%$

	0	1	2	3	...	27	28
B_t:	2800	2700	2600	2500	...	100	0
I_t:		70	67.50	65	...	5	2.5
P_t:		100	100	100	...	100	100

$$2800(1+i)^{14} = 170\, s_{\overline{28|}k} - 2.5\,\frac{s_{\overline{28|}k} - 28}{k}, \quad k = 3\%$$

$$i = \boxed{5.662\%}$$

5. Brown is repaying a $1120 loan with 8 equal payments of principal at the end of each year. Interest at an annual effective rate of 10.5% is paid on the outstanding balance each year. The first installment is due one year from today. Immediately after the loan was made, the loan was sold to an investor. Find the price the investor paid to earn an annual effective rate of 6.5%. [6.f #06]

$\boxed{\text{A)}}$ At least $1,280, but less than $1,300
B) At least $1,240, but less than $1,260
C) At least $1,260, but less than $1,280
D) At least $1,300, but less than $1,320
E) At least $1,320, but less than $1,340

	0	1	2	3	...	7	8
B_t:	1120	980	840	700	---	140	0
I_t:		117.6	102.9	88.2	---	29.4	14.7
P_t:		140	140	140	---	140	140

$$P = 257.6\, a_{\overline{8}|6.5\%} - 14.7\, \frac{a_{\overline{8}|6.5\%} - 8v^8}{0.065}$$

$$= \boxed{1284.66}$$