

## HW 4.1 (b) Key

1. A 1000 bond with coupon rate  $c$  convertible semiannually will be redeemed at par in  $n$  years. The purchase price to yield 9% convertible semiannually is  $P$ . If the coupon rate were  $c - 0.015$ , the price of the bond would be  $P - 125.92$ . Another 1000 bond is redeemable at par at the end of  $2n$  years. It has a coupon rate of 9.5% convertible semiannually and the yield rate is 8% convertible semiannually. Calculate the price of this second bond. [7.a-b #21]

(A) 1170 B) 1110 C) 1235 D) 1300 E) 1365

$$\begin{aligned} \text{coupon rate: } r &= \frac{c}{2} & P &= 1000 r a_{\overline{2n}|4.5\%} + 1000 v^{2n} \\ \text{coupon rate: } \frac{1}{2}(c - 0.015) & & P - 125.92 &= 1000(r - 0.0075) a_{\overline{2n}|4.5\%} + 1000 v^{2n} \\ 125.92 &= 7.5 a_{\overline{2n}|4.5\%} \rightarrow 2n = 32 \rightarrow n = 16 \end{aligned}$$

$$\text{Bond 2: } P = 47.5 a_{\overline{64}|4\%} + 1000 v^{64} = \boxed{1172.26}$$

2. An insurance company owns a 1000 par value 9% bond with semiannual coupons. The bond will mature for 1000 at the end of 11 years. The company decides that an 6-year bond would be preferable. Current yield rates are 7.5% compounded semiannually. The company uses the proceeds from the sale of the 9% bond to purchase a 5.5% bond with semiannual coupons, maturing at par at the end of 6 years. Calculate the par value of the 6-year bond. [7.a-b #24]

(A) 1228 B) 1244 C) 1260 D) 1276 E) 1292

$$\text{Current value of 11 yr bond: } P = 45 a_{\overline{22}|3.75\%} + 1000 v^{22} = 1111.02$$

11 yr bond sold for 1111.02, which is used to purchase 6 yr bond.

$$1111.02 = 0.0275 F a_{\overline{12}|3.75\%} + F v^{12} \rightarrow 1111.02 = F(0.90477) \\ F = \boxed{1227.95}$$

3. Two bonds are both redeemable at their par value of 100 in  $t$  years. Bond A has 5.5% semiannual coupons and costs 82. Bond B has 6.5% semiannual coupons and costs 88. The bonds were purchased to produce the same yield rate. What is the yield rate per annum convertible semiannually? [7.a-b #25]

- (A) At least 8.4%, but less than 8.6% D) At least 8.6%, but less than 8.8%  
 (B) At least 8%, but less than 8.2% E) At least 8.8%, but less than 9%  
 (C) At least 8.2%, but less than 8.4%

$$A: 82 = 2.75 a_{\overline{n}|i} + 100 v^n$$

$$B: 88 = 3.25 a_{\overline{n}|i} + 100 v^n$$

↓

$$6 = 0.5 a_{\overline{n}|i}$$

$$a_{\overline{n}|i} = 12$$

$$A: 82 = 2.75(12) + 100 v^n$$

$$v^n = 0.49$$

$$a_{\overline{n}|i} = 12 \rightarrow \frac{1 - 0.49}{i} = 12 \rightarrow i = 4.25\% \rightarrow i^{(2)} = \boxed{8.5\%}$$

4. A bond with coupons equal to 35 sells for 1000. A second bond with the same maturity value and term has coupons equal to 20 and sells for 800. A third bond with the same maturity value and term has coupons equal to 90. All prices are based on the same yield rate, and all coupons are paid at the same frequency. Determine the price of the third bond. [7.a-b #27]

(A) 1730   B) 1790   C) 1850   D) 1900   E) 1960

$$\begin{aligned}
 \textcircled{1} \quad 1000 &= 35a_{\overline{n}|i} + Xv^n \\
 \textcircled{2} \quad 800 &= 20a_{\overline{n}|i} + Xv^n \quad \left. \begin{array}{l} \textcircled{1} \\ \textcircled{2} \end{array} \right\} \rightarrow 200 = 15a_{\overline{n}|i} \rightarrow a_{\overline{n}|i} = 13.3333 \\
 &\hspace{15em} Xv^n = 533.3333 \\
 \textcircled{3} \quad P &= 90a_{\overline{n}|i} + Xv^n = 90(13.3333) + 533.3333 \\
 &= \boxed{1733.33}
 \end{aligned}$$

5. Two bonds are purchased for the same price to yield 4%. Bond X has 3% annual coupons and matures for its face value of 100. Bond Y has annual coupons of 2 and matures for 170. Both bonds mature at the end of  $n$  years. Calculate  $n$ . [7.a-b #28]

(A) 34   B) 20   C) 24   D) 27   E) 31

$$\begin{aligned}
 X: \quad P &= 3a_{\overline{n}|4\%} + 100v^n \\
 Y: \quad P &= 2a_{\overline{n}|4\%} + 170v^n \quad \left. \begin{array}{l} X \\ Y \end{array} \right\} \rightarrow 0 = a_{\overline{n}|4\%} - 70v^n \\
 &\hspace{15em} 70v^n = \frac{1-v^n}{0.04} \\
 v^n &= 0.2632 \\
 n &= \frac{\ln(0.2632)}{\ln(1.04^{-1})} = \boxed{34}
 \end{aligned}$$