HW 4.1 (d) Key

1. Bill buys a 22-year 1000 par value 7% bond with semiannual coupons. The price assume a nominal yield of 7%, compounded semiannually. As Bill receives each coupon payment, he immediately puts the money into an account earning interest at an annual effective rate of *i*. At the end of 22 years, immediately after Bill receives the final coupon payment and the redemption value of the bond, Bill has earned an annual effective yield of 5.95% on his investment in the bond. Calculate *i*. [7.a-b #02]

A) 4.5% B) 4% C) 4.25% D) 4.75% E) 5%

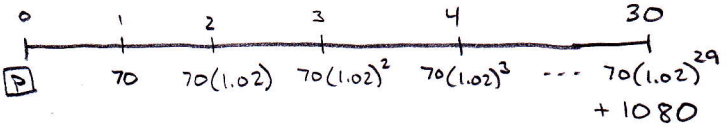
$F = C = 1000 \quad r = 3.5\% \quad Fr = 35 \quad n = 44$

yield rate = $k = 3.5\%$ $\quad r = k$ and $F = C$, so $P = F = 1000$.

$1000(1.0595)^{22} = 35\, s_{\overline{44}|j} + 1000 \rightarrow j = 2.2202\% \rightarrow \boxed{i = 4.49\%}$

2. A 1000 par value 30-year bond with annual coupons and redeemable at maturity at 1080 is purchased for *P* to yield an annual effective rate of 8%. The first coupon is 70. Each subsequent coupon is 2% greater than the preceding coupon. Determine *P*. [7.a-b #03]

A) 1065 B) 990 C) 1025 D) 1100 E) 1140



$i = 8\% \quad k = 2\%$

$i' = \frac{i-k}{1+k} = \frac{0.06}{1.02} = 5.882\%$

$P = \frac{70}{1.02} a_{\overline{30}|i'} + 1080(1.08)^{-30} = \boxed{1063.98}$

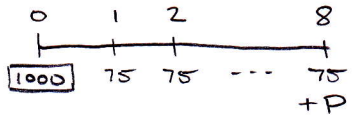
3. Tina buys a 1000 par value 14-year bond with 7.5% annual coupons at a price to yield an annual effective rate of 7.5%. The coupons were reinvested at an annual effective rate of 5%. Immediately after receiving payment number 8, Tina sells the bond to Joe for a price of *P*. *P* assumes an annual effective yield of *i* to the buyer. Tina's annual effective yield from the date of purchase until the date of sale was 5%. Calculate *i*. [7.a-b #06]

A) 13.6% B) 12.8% C) 13% D) 13.3% E) 13.8%

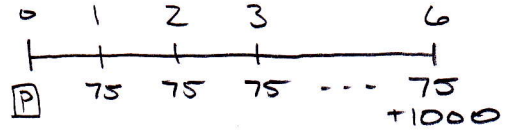
$F = C = 1000 \quad r = j = 7.5\% \quad n = 14 \quad r = j$ and $F = C$, so $P = F = 1000$. (yield rate)

Coupons $= Fr = 75$

Tina: (5%)



Joe: (i)



$1000 = 75 a_{\overline{8}|5\%} + P v^8$

$P = 761.27$

$761.27 = 75 a_{\overline{6}|i} + 1000 v^6$

$\boxed{i = 13.57\%}$

4. John purchases a 1000 par value 18-year bond with coupons at 5% convertible semiannually which will be redeemed for R. The purchase price is 800 and the present value of the redemption value is 292.74. Calculate R. [7.a-b #07]

[A] 1010 B) 980 C) 1040 D) 1070 E) 1100

$F = 1000 \quad r = 2.5\% \quad n = 36 \quad P = 800 \quad Rv^{36} = 292.74$

$800 = 25 a_{\overline{36}|i} + 292.74 \rightarrow i = 3.5\%$

$R = 292.74 (1.035)^{36} = \boxed{1010}$

5. Two 1000 par value bonds are purchased. The $2n$-year bond costs 120 more than the n-year bond. Each has 12% annual coupons and each is purchased to yield 8% annual effective. Calculate the price of the n-year bond. [7.a-b #08]

[A] 1200 B) 1080 C) 1140 D) 1260 E) 1320

$F = 1000 \quad r = 12\% \quad i = 8\%$

$P + 120 = 120 a_{\overline{2n}|} + 1000 v^{2n}$

$P = 120 a_{\overline{n}|} + 1000 v^{n}$

$P = 120 \frac{1 - 0.6}{0.08} + 1000(0.6)$

$= \boxed{1200}$

$120 a_{\overline{n}|} + 1000 v^{n} + 120 = 120 a_{\overline{2n}|} + 1000 v^{2n}$

$\downarrow \times 0.08$

$120(1 - v^{n}) + 80 v^{n} + 9.6 = 120(1 - v^{2n}) + 80 v^{2n}$

$\downarrow$

$40 v^{2n} - 40 v^{n} + 9.6 = 0$

$\downarrow$

$v^{n} = 0.6$