HW 4.2 (b) Key

$$n = 24$$

1. Among a company's assets and accounting records, an actuary finds a 12-year bond that was purchased at a premium. From the records, the actuary has determined the following:
   (i) The bond pays semiannual interest.
   (ii) The amount for amortization of the premium in coupon payment number 4 was 855.68.
   (iii) The amount for amortization of the premium in coupon payment number 9 was 968.12.
   What is the value of the premium? [7.c #02]

   A) 25,704   B) 21,386   C) 22,465   D) 23,545   E) 24,624

   $$P_9 = P_4 (1+i)^5 \longrightarrow 968.12 = 855.68 (1+i)^5 \longrightarrow i = 2.5\%$$

   $$P_1 = P_4 v^3 = 855.68 (1.025)^{-3} = 794.58$$

   $$\text{Total Premium} = P_1 + P_2 + \dots + P_{24} = P_1 (1 + 1.025 + 1.025^2 + \dots + 1.025^{23})$$

   $$= P_1 \, s_{\overline{24}|\,2.5\%} = \boxed{25,704.03}$$

2. Becky buys an n-year 1000 par value bond with 5.5% annual coupons at a price of 877.58. The price assumes an annual effective yield rate of i. The total write-up in the book value of the bond during the first 2 years after purchase is 22.45. Calculate i. [7.c #08]

   A) 7.5%   B) 7.7%   C) 7.9%   D) 8.11%   E) 8.31%

   $$877.58 = 55 \, a_{\overline{n}|i} + 1000 \, v^n$$

   Found using BA II or quad formula.

   $$877.58 = 55 \, a_{\overline{2}|i} + (877.58 + 22.45) \, v^2 \longrightarrow \boxed{i = 7.5\%}$$

3. An n-year 1000 par value bond with 6.5% annual coupons has an annual effective yield of i, $i > 0$. The book value of the bond at the end of year 8 is 1224.68 and the book value at the end of year 10 is 1204.46. Calculate the purchase price of the bond. [7.c #09]

   A) 1290   B) 1255   C) 1267   D) 1278   E) 1302

   $$P = 65 \, a_{\overline{n}|i} + 1000 \, v^n$$

   $$B_8 = 1224.68 \qquad B_{10} = 1204.46$$

   $$B_8 = 65 \, a_{\overline{2}|i} + B_{10} \, v^2 \longrightarrow i = 4.5\% \ (\text{using BA II})$$

   $$P = 65 \, a_{\overline{8}|i} + B_8 \, v^8 = \boxed{1289.91} \ (\text{using BA II})$$

4. A 28-year 10,000 bond that pays 4% annual coupons matures at par. It is purchased to yield 5.5% for the first 19 years, and 4.5% thereafter. Calculate the amount for accumulation of discount for year 6. [7.c #14]

A) 61   B) 58   C) 65   D) 68   E) 71

$$P = 400\, a_{\overline{19}|\,5.5\%} + 400\, a_{\overline{9}|\,4.5\%}(1.055)^{-19} + 10{,}000\,(1.045)^{-9}\,(1.055)^{-19}$$

$$= 8127.44$$

$$B_5 = 8127.44\,(1.055)^5 - 400\, s_{\overline{5}|\,5.5\%} = 8389.80$$

$$I_6 = 0.055 \cdot B_5 = 461.44$$

$$P_6 = 461.44 - 400 = \boxed{61.44}$$

5. A 1000 par value bond with 6.5% annual coupons is purchased at a discount 22 years prior to the maturity date. The proceeds of the coupons are invested in a savings account with a 9.5% effective annual rate of interest. The effective yield on the 22 year investment - including the bond and the savings account - is 9.61%. What is the book value of the bond one year after purchase? [7.c #19]

A) 716   B) 692   C) 700   D) 708   E) 724

$$P = 65\, a_{\overline{22}|\,i} + 1000\, v^n$$

$$P\,(1.0961)^{22} = 65\, s_{\overline{22}|\,9.5\%} + 1000 \quad\rightarrow\quad P = 711.21$$

$$i = 9.7282 \quad (\text{using BA II})$$

$$B_1 = P(1+i) - 65 = \boxed{715.40}$$

↑

Small rounding
issue somewhere.