HW 4.5 (b) Key

(This space for rent.)

1. You are given the following information about ~~three zero-coupon~~ four bonds:

Term in Years	Coupon Rate	Yield-to-Maturity	Price
0.5	0%	4%	98.04
1.0	0%	4.4%	95.74
1.5	5%	4.8%	100.29
2.0	6%	5.2%	101.50

F=100 for all four bonds

All rates are expressed as nominal annual rates of interest convertible semiannually. Coupons are paid semiannually. Determine the spot rate for a 1.5-year period, expressed as a nominal annual rate of interest convertible semiannually. [9.a-b #09]

A) 4.81%   B) 4.99%   C) 5.17%   D) 5.35%   E) 5.53%

$S_1 = 2\%$ $S_2 = 2.2\%$

$$100.29 = \frac{2.5}{1+S_1} + \frac{2.5}{(1+S_2)^2} + \frac{102.5}{(1+S_3)^2} \rightarrow S_3 = 2.4053\%$$

$$\boxed{4.816\%}$$

2. The one-year spot rate is 2.5%. A two-year $100 bond maturing at par, with 4.5% annual coupons, is currently selling for its par value. Determine the two-year spot rate.   [9.a-b #10]
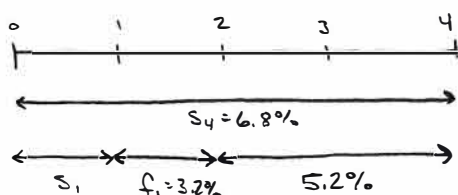
   A) 4.55%   B) 4.50%   C) 4.59%   D) 4.64%   E) 4.68%

$$S_1 = 2.5\%$$

$$100 = \frac{4.5}{1+S_1} + \frac{104.5}{(1+S_2)^2} \quad\rightarrow\quad S_2 = \boxed{4.546\%}$$

3. The one-year forward rate for year 2 (i.e., the one-year effective rate during year 2) is 3.2%.   The four-year spot rate is 6.8%.   The expected spot rate at the end of year two on a zero-coupon bond bond maturing at the end of year 4 is 5.2%.   Determine the one-year spot rate.   [9.a-b #11]

   A) 13.91%   B) 10.91%   C) 11.66%   D) 12.41%   E) 13.16%

$$(1.068)^4 = (1+S_1)(1.032)(1.052)^2$$

$$S_1 = \boxed{13.91\%}$$

4. You are given the following information about two bonds that will mature in four years at par:

	Bond A	Bond B
Par Value	$800	$1000
Annual Coupon Rate	4%	2%
Price	$660	$820

Determine the four-year spot rate.   [9.a-b #12]

   A) 5.25%   B) 4.40%   C) 4.68%   D) 4.96%   E) 5.53%

$$A: \quad 660 = \frac{32}{1+S_1} + \frac{32}{(1+S_2)^2} + \frac{32}{(1+S_3)^3} + \frac{832}{(1+S_4)^4}$$

$$B: \quad 820 = \frac{20}{1+S_1} + \frac{20}{(1+S_2)^2} + \frac{20}{(1+S_3)^3} + \frac{1020}{(1+S_4)^4}$$

$$1.6\,(Eqn\ B) - Eqn\ A \quad gives: \quad 652 = \frac{800}{(1+S_4)^4} \quad\rightarrow\quad S_4 = \boxed{5.247\%}$$

5. A 1000 par value bond with 7% annual coupons matures at par in two years.   You are given that the one-year spot rate is 6% and the one-year forward rate for year 2 (i.e., the one-year effective rate during year 2) is 8.5%. Determine the price of the bond.   [9.a-b #15]

   A) 996.39   B) 834.98   C) 888.78   D) 942.59   E) 1050.20

$$S_1 = 6\% \qquad f_1 = 8.5\%$$

$$(1+S_2)^2 = (1.06)(1.085) \quad\rightarrow\quad S_2 = 7.2427\%$$

$$P = \frac{70}{1+S_1} + \frac{1070}{(1+S_2)^2} = \boxed{996.39}$$