

HW 5.1 Key

1. Consider a 3-year loan. The lender requires a real risk-free rate of 5.2% for the deferral of consumption. Based on credit history, the lender estimated that the borrower has a 10% chance of defaulting. In the case of a default, the lender expects to be able to recuperate 20% of the repayment amount. The lender predicts that the inflation rate of the next 3 years will be 1.7%. Determine the continuously compounded rate of interest that should be set by the lender.

A) 9.68% B) 9.49% C) 9.87% D) 10.07% E) 10.26%

$$e^{3(0.052 + 0.017)} = 0.9e^{3R} + 0.1(0.2)e^{3R}$$

$$e^{0.207} = 0.92e^{3R}$$

$$R = \boxed{0.09679}$$

2. Consider a 6-year loan. The lender requires a real risk-free rate of 4.8% for the deferral of consumption. Based on credit history, the lender estimated that the borrower has a $p\%$ chance of defaulting. The lender sets a continuously compounded interest rate of 7.5% for the loan. Assuming that the effects of inflation were ignored, find p .

A) 14.96% B) 15.55% C) 16.15% D) 16.75% E) 17.35%

$$e^{6(0.048)} = (1-p)e^{6(0.075)}$$

$$1-p = 0.8504$$

$$p = \boxed{0.1496}$$

3. Consider a 7-year loan. The lender requires a real risk-free rate of 6% for the deferral of consumption. The loan agreement provide the lender with inflation protection, with the inflation adjustment determined by the percentage increase in the CPI during the term of the loan. The CPI is equal is 238.39 when the loan is entered into, and is equal to 263.3 when the loan is repaid. Assume that there is no risk of default. Determine the continuously compounded rate of interest for the loan.

A) 7.42% B) 7.12% C) 7.27% D) 7.57% E) 7.72%

$$e^{7i_a} = \frac{263.3}{238.39} \Rightarrow i_a = 0.01420$$

$$R = r + i_a = 0.06 + 0.01420$$

$$= \boxed{0.07420}$$

4. Consider a 4-year loan. The lender requires a real risk-free rate of 5.2% for the deferral of consumption. Based on credit history, the lender estimated that the borrower has a 14% chance of defaulting. In the case of a default, the lender expects to be able to recuperate 25% of the repayment amount. The loan agreement includes inflation protection for the lender. The actual continuously compounded rates of inflation for the four years during the term of the loan are 2.2%, 0.9%, 1.9%, and 1.8%. Assuming that the borrower did not default, determine the continuously compounded rate of interest actually paid to the lender.

A) 9.67% B) 8.90% C) 9.09% D) 9.29% E) 9.48%

$$e^{4i_a} = e^{0.022} e^{0.009} e^{0.019} e^{0.018}$$

$$i_a = \frac{1}{4}(0.022 + 0.009 + 0.019 + 0.018) = 0.017$$

$$e^{4(0.052 + 0.017)} = 0.86 e^{4R} + 0.14(0.25) e^{4R}$$

$$e^{0.276} = 0.895 e^{4R}$$

$$R = \boxed{0.09673}$$

5. Consider a 4-year loan in the amount of 1000. Based on credit history, the lender estimated that the borrower has a 14% chance of defaulting. In the case of a default, the lender expects to be able to recuperate 10% of the repayment amount. The loan agreement includes inflation protection for the lender. The actual continuously compounded rates of inflation for the four years during the term of the loan are 2.5%, 2.1%, 1%, and 1.4%. The borrower did not default, and repaid the lender an amount of 1372.56 when the loan matured. Determine the continuously compounded rate of interest required by the lender for the deferral of consumption.

A) 2.80% B) 2.58% C) 2.69% D) 2.91% E) 3.02%

$$e^{4i_a} = e^{0.025} e^{0.021} e^{0.01} e^{0.014}$$

$$i_a = \frac{1}{4}(0.025 + 0.021 + 0.01 + 0.014) = 0.0175$$

$$1000 e^{4(r + 0.0175)} = 0.86(1372.56) + 0.14(137.256)$$

$$r = \boxed{0.028}$$

Note:

$$1000 e^{4R} = 1372.56 \Rightarrow R = 0.07917$$

$$R = r + i_a + s \Rightarrow 0.07917 = 0.028 + 0.0175 + s$$

$$\Rightarrow s = 0.03367$$