HW 5.3 - 5.4 (b) Key

1. On January 1, an investment account is worth 220,000. M months later, the value has increased to 228,800 and 8,800 is withdrawn. $2M$ months prior to the end of the year, the account is again worth 228,800 and 8,800 is withdrawn. On December 31, the account is worth 228,800. The annual effective yield rate, using the dollar-weighted method, is 12.55%. Calculate M. [5.d-e #11]

A) 1.15 B) 1.25 C) 1.5 D) 1.75 E) 2

	(1/1)	(M+1/1)	(13-2M/1)	(12/31)
t:	0	$\frac{M}{12}$	$\frac{12-2M}{12}$	1
Beg:	0	228.8	228.8	228.8
Trans:	+220	-8.8	-8.8	-228.8
End:	220	220	220	0

$$220\left(1+0.1255(1)\right) - 8.8\left(1+0.1255\,\tfrac{12-M}{12}\right) - 8.8\left(1+0.1255\,\tfrac{2M}{12}\right) - 228.8 = 0$$

$$247.61 - 8.8 - 0.09203(12-M) - 8.8 - 0.09203(2M) - 228.8 = 0$$

$$0.09203M = 0.1056$$

$$\boxed{M = 1.1474}$$

2. You are given the following information about an investment account:

	1/1/2000	3/1/2000	4/1/2000	*T*/1/2000	1/1/2
Account Value (Before Transaction)	100	108	96	102	126
Deposit			12	*X*	
Withdrawal		20			

The time-weighted yield rate is 18.82%, and the dollar-weighted yield rate is 18.18%. Calculate *T*. [5.d-e #12]

[A] 8 B) 6 C) 7 D) 9 E) 10

k months

	(1/1)	(3/1)	(4/1)	(T/1)	(1/1)
t	0	2/12	3/12	(T-1)/12	1
Beg:	0	108	96	102	126
Trans:	+100	-20	+12	+X	-126
End:	100	88	108	102+X	0

$$\text{TWR: } 0.1882 = \frac{108}{100}\,\frac{96}{88}\,\frac{102}{108}\,\frac{126}{102+X} - 1 \rightarrow X = 16$$

$$\text{DWR: } 0.1818 = \frac{18}{100(1) - 20\left(\frac{5}{6}\right) + 12\left(\frac{3}{4}\right) + 16\left(\frac{k}{12}\right)} \rightarrow k = 5 \rightarrow T - 1 = 7 \rightarrow \boxed{T = 8}$$

3. 200 is deposited into a fund on January 1, 2010. Another deposit is made into the fund on July 1, 2010. On January 1, 2011, the balance in the fund is 460. The time-weighted yield rate is 11% and the dollar-weighted yield rate is 5%. Calculate the annual effective interest rate earned on the fund during the first six months of 2010. [5.d-e #14]

[A] ~~19.8%~~ 29.4% B) 15.1% C) 16.5% D) 17.9% E) 19.3%

	(1/1)	(7/1)	(1/1)
t	0	1/2	1
Beg:	0	X	460
Trans:	+200	+D	-460
End:	200	X+D	0

$$\text{DWR: } 0.05 = \frac{260 - D}{200 + 0.5D} \rightarrow D = 243.90$$

$$\text{TWR: } 0.11 = \frac{X}{200}\,\frac{460}{X + 243.90} - 1 \rightarrow X = 227.51$$

$$i = \left(\frac{X}{200}\right)^2 - 1 = \boxed{29.4\%}$$

4. You are given the following table of interest rates:

Calendar Year of Original Investment	Investment Year Rates (in %)			Portfolio Rates (in %)	Calendar Year of Portfolio Rate
y	i_1^y	i_2^y	i_3^y	i^{y+5}	
2004	5.5	5.9	4.3	4.3	2007
2005	4	5.9	5.2	5	2008
2006	5.4	4.7	5.7	5.1	2009
2007	4.8	4	4.1	4.6	2010
2008	5	5.3	4.2	5.4	2011
2009	5.5	5.9	4.2	5.1	2012
2010	6	5.9	4.6	5.7	2013
2011	4.6	4.2	4.5		
2012	4	4.3			
2013	4.5				

A person deposits 1000 on January 1, 2007 Let the following be the accumulated value of the 1000 on January 1, 2013:

P: under the investment year method
Q: under the portfolio yield method
R: where the balance is withdrawn at the end of every year and is reinvested at the new money rate

5. Determine the sum of *P*, *Q*, and *R*. [5.f #01]

A) 3,987 B) 3,907 C) 3,947 D) 4,027 E) 4,067

$P = 1000(1.048)(1.04)(1.041)(1.046)(1.054)(1.051) = 1314.68$

$Q = 1000(1.043)(1.05)(1.051)(1.046)(1.054)(1.051) = 1333.70$

$R = 1000(1.048)(1.05)(1.055)(1.06)(1.046)(1.04) = 1338.67$

3987.03

The following table shows the annual effective interest rates being credited by an investment account, by the calendar year of investment. The investment year method is applicable for the first 3 year, after which a portfolio rate is used:

Calendar Year of Original Investment	Investment Year Rates (in %)			Portfolio Rates (in %)	Calendar Year of Portfolio Rate
y	i_1^y	i_2^y	i_3^y	i^{y+5}	
2006	5	8	t	8	2009
2007	12	10	10	7	2010
2008	10	$t-4$	8	12	2011
2009	5	12	11	10	2012
2010	5	6	10	5	2013

An investment of 100 is made at the beginning of years 2006, 2007, and 2008. The total amount of interest credited by the fund during year 2009 is equal to 27.71. Calculate t. [5.f #02]

A) 9 B) 8.25 C) 8.5 D) 8.75 E) 9.25

$100(1.05)(1.08)(1+t)(0.08) + 100(1.12)(1.10)(0.10) + 100(1.10)(t-0.04) = 27.71$

$9.072(1+t) + 12.32 + 110(t-0.04) = 27.71$

$t = 9\%$