HW 6.2 Key

1. An annuity-immediate has payments of \$4500, \$5000, and \$3000 at the end of years 2, 4, and 6, respectively. Determine the convexity of the payments evaluated at $i = 5\%$. [9.i-n #01]

[A] 17.45 B) 16.76 C) 18.15 D) 18.85 E) 19.55

$$P = 4.5v^2 + 5v^4 + 3v^6 = 10.4338$$

$$-P' = 9v^3 + 20v^5 + 18v^7$$

$$P'' = 27v^4 + 100v^6 + 126v^8 = 182.1163$$

$$\text{Conv} = \frac{P''}{P} = \boxed{17.45}$$

2. An investment will return \$4000 in 3 years and \$1000 in 7 years. Determine the ratio of the convexity of the payments to their modified duration, evaluated at $i = 8\%$. [9.i-n #03]

[A] 4.82 B) 4.04 C) 4.24 D) 4.43 E) 4.62

$$P = 4v^3 + v^7 = 3.7588$$

$$-P' = 12v^4 + 7v^8 = 12.6022$$

$$P'' = 48v^5 + 56v^9 = 60.6819$$

$$\frac{\text{Conv}}{\text{Mod D}} = \frac{P''/P}{-P'/P} = \frac{P''}{P'} = \boxed{4.815}$$

3. Find the convexity of a 4-year bond paying annual coupons of 4% and yielding 5.5% annually. [09i_B3]

[A] 16.59 B) 15.09 C) 15.59 D) 16.09 E) 17.08

$$P = 4v + 4v^2 + 4v^3 + 104v^4 = 94.7423$$

$$-P' = 4v^2 + 8v^3 + 12v^4 + 416v^5$$

$$P'' = 8v^3 + 24v^4 + 48v^5 + 2080v^6 = 1571.4239$$

$$\text{Conv} = \frac{P''}{P} = \boxed{16.59}$$

4. Find the convexity of a perpetuity that makes annual payments of 20. Assume an effective annual rate of 3.5%. [09i_B4]

[A] 1633 B) 1559 C) 1584 D) 1608 E) 1657

$$P = \frac{20}{i} = 571.4286$$

$$P' = -\frac{20}{i^2}$$

$$P'' = \frac{40}{i^3} = 932{,}944.6064$$

$$Conv = \frac{P''}{P'} = \boxed{1632.65}$$

5. A perpetuity makes payments at the end of each year. The first payment is in the amount of 19 and each subsequent payment increases by 2. Assuming an effective annual interest rate of 4.5%, find the convexity of this perpetuity. [09i_B5]

[A] 2371 B) 2229 C) 2265 D) 2300 E) 2336

$$P = \frac{19}{i} + \frac{2}{i^2} = 1409.8765$$

$$P' = -\frac{19}{i^2} - \frac{4}{i^3}$$

$$P'' = \frac{38}{i^3} + \frac{12}{i^4} = 3{,}343{,}392.775$$

$$Conv = \frac{P''}{P'} = \boxed{2371.41}$$