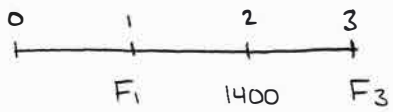
HW 6.3 (a) Key

1. A company must pay a benefit of 1400 to a customer in two years. To provide for the benefit, the company will buy one-year and three-year zero-coupon bonds. The one-year and three-year spot rates are 4% and 5% respectively. The company wants to fully immunize itself from changes in market rate, which is currently at 10%. What amount should it invest in the one-year bonds? [9.i-n #05]

A) 612 B) 567 C) 589 D) 635 E) 657



$i = 10\%$, $S_1 = 4\%$, $S_2 = 5\%$

Note: The spot rates in this problem are used only to price the individual bonds.

i) $F_1 v + F_3 v^3 = 1400 v^2$

ii) $F_1 v^2 + 3F_3 v^4 = 2800 v^3$

$\rightarrow$ $3F_1 + 3F_3 v^2 = 4200 v$

$F_1 + 3F_3 v^2 = 2800 v$

$2F_1 = 1400 v$

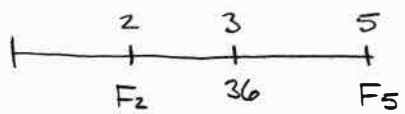
$F_1 = 636.36$

$F_1/(1+S_1) = \boxed{611.89}$

2. A company has a liability of 36,000 in 3 years. The company plans to fully immunize this liability using a combination of 2-year zero-coupon bonds and 5-year zero-coupon bonds.

Assuming an annual effective interest rate of 4%, find the par value of the 2-year zero-coupon bond. [09i_A5]

A) 23,077 B) 22,154 C) 22,615 D) 23,538 E) 24,000



$i = 4\%$

i) $F_2 v^2 + F_5 v^5 = 36 v^3$

ii) $2F_2 v^3 + 5F_5 v^6 = 108 v^4$

$\downarrow$

$5F_2 + 5F_5 v^3 = 180 v$

$2F_2 + 5F_5 v^3 = 108 v$

$\downarrow$

$3F_2 = 72 v$

$F_2 = 24v = \boxed{23.077}$

3. Company ABC is required to pay their customers an amount of 15,000 after 3.5 years and 55,000 after 6.5 years. Based on an annual effective interest rate of 6%, the company's actuary uses duration matching to construct a portfolio of assets consisting of the following:

i) 3-year bond with 4% semiannual coupons.
ii) 7.5-year zero-coupon bond.

$i = 6\%$.

Calculate the amount invested in the 7.5-year zero-coupon bond. [09i_A2]

A) 31,262 B) 26,010 C) 27,761 D) 29,512 E) 33,013

Let's look at the liabilities first:

$P = 15v^{3.5} + 55v^{6.5} = 49.89215$

$P' = 52.5v^{4.5} + 357.5v^{7.5} = 271.3217$

$\text{ModD}_L = 5.438165$

Duration of Assets:

i) For simplicity, assume a par value of 100.
This isn't the true par value, but the duration will be the same.

$P = 2v^{0.5} + 2v^{1} + 2v^{1.5} + 2v^{2} + 2v^{2.5} + 102v^{3} = 94.8120$

$P' = v^{1.5} + 2v^{2} + 3v^{2.5} + 4v^{3} + 5v^{3.5} + 306v^{4} = 255.1063$

$\text{ModD}_i = 2.6907$

ii) $\text{Mod D}_{ii} = 7.5v = 7.07547$

Let X = (Price of 7.5-year bond) ÷ (Total price of Assets). Then:

$\text{ModD}_A = (1-X)\text{ModD}_i + X\,\text{ModD}_{ii} = 2.6907(1-X) + 7.0755X$

$\text{ModD}_A = \text{ModD}_L \Rightarrow 2.6907(1-x) + 7.0755X = 5.4382$

$X = 0.6267$

$P_A = P_L = 49.892$

$P_{ii} = X P_L = 0.6267(49.892) = \boxed{31.266}$

4. Company ABC is required to pay their customers 22,000 after 2 years. The company uses two of the following bonds to construct a portfolio that fully immunizes the potential interest rate risk.

(i) 1-year zero-coupon bond
~~(ii) 2-year zero-coupon bond~~
(iii) 3-year zero-coupon bond
(iv) 4-year zero-coupon bond

$i = 8\%$

The market interest rate is 8%. Of the combinations that achieve full immunization, the company purchases the two bonds that have the smallest total par value. Find the total par value of these two bonds. [09i_A3]

A) 22,065 B) 22,134 C) 22,000 D) 22,206 E) 21,859

(i & iii)

$$F_1 v + F_3 v^3 = 22 v^2$$
$$F_1 v^2 + 3F_3 v^4 = 44 v^3$$
$$\downarrow$$
$$F_1 + F_3 v^2 = 22v$$
$$F_1 + 3F_3 v^2 = 44v$$
$$\downarrow$$
$$2F_3 v^2 = 22v$$

$F_3 = 11.88$, $F_1 = 10.185$ → **22.065**

(i & iv)

$$F_1 v + F_4 v^4 = 22 v^2$$
$$F_1 v^2 + 4F_4 v^5 = 44 v^3$$
$$\downarrow$$
$$F_1 + F_4 v^3 = 22v$$
$$F_1 + 4F_4 v^3 = 44v$$
$$\downarrow$$
$$3F_4 v^3 = 22v$$

$F_4 = 8.554$, $F_1 = 13.580$ → 22.134

5. A company has a liability of 600 in 1 year and a liability of 400 in 5 years. To pay for these liabilities, the company purchases a 2-year zero coupon bond and a 3-year zero coupon bond. The company purchases the bonds to ensure duration matching (that is, to satisfy the first two immunization conditions), assuming a market rate of 3%.

i) Determine the par value of the 2-year bond.
ii) Has Redington immunization been achieved?

A) 504; No B) 523; No C) 504; Yes D) 523; Yes E) There is not enough information to determine.

i) $F_2 v^2 + F_3 v^3 = 6v + 4v^5$
ii) $2F_2 v^3 + 3F_3 v^4 = 6v^2 + 20v^6$

→ $F_2 = \boxed{5.04}$, $F_3 = 4.95$

iii) $P_A'' = 6F_2 v^4 + 12F_3 v^5 = 78.107$

$P_L'' = 12v^3 + 120v^7 = 108.553$

$P_A'' < P_L''$. **Portfolio is NOT immunized.**