HW 6.3 (b) Key

1. A company must pay liabilities of 1200 due one year from now and another 2400 due two years from now. There are two available investments: one-year zero coupon bonds and two-year bonds with 6% annual coupons maturing at par. The one year spot rate is 4% and the one year forward rate is 5%. What is the company's total cost of the bonds required to exactly (absolutely) match the liabilities? [9.i-n #04]

   A) 3352 B) 2856 C) 2980 D) 3104 E) 3228

$s_1 = 4\%$ $f_1 = 5\%$

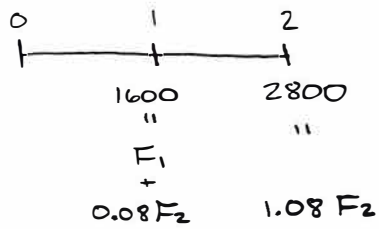
We could find the amount invested in each bond, but we don't need to in this problem. We know the bonds have to pay a total of 1200 at $t=1$, and 2400 at $t=2$. We simply need to find the PV of these bond payments:

$$PV = \frac{1200}{1.04} + \frac{2400}{(1.04)(1.05)} = \boxed{3351.65}$$

2. A company must pay liabilities of 1600 due one year from now and another 2800 due two years from now. There are two available investments: one-year zero coupon bonds and two-year bonds with 8% annual coupons maturing at par. The one year spot rate is 3% and the one year forward rate is 5%. The company plans to use bonds to exactly (absolutely) match the liabilities. How much must they invest in one-year bonds? [9.i-n #04]

   A) 1352 B) 1302 C) 1402 D) 1452 E) 1502

$s_1 = 3\%$ $f_1 = 5\%$



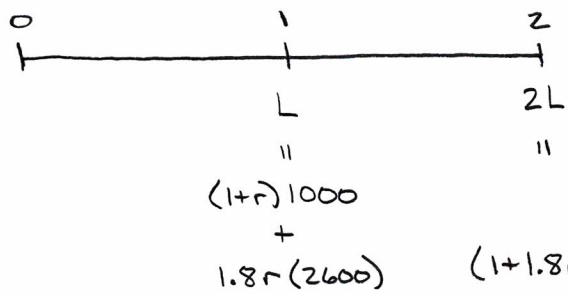
$\rightarrow$ $F_2 = 2592.59$

$F_1 = 1392.59$

$$P_1 = \frac{F_1}{1.03} = \boxed{1352.03}$$

3. A company must pay a liability of L due one year from now and $2L$ due two years from now. The company exactly (absolutely) matches the liabilities by buying a one-year bond with face value 1000 and a two-year bond with face value 2600. Both bonds have annual coupons, with a coupon rate of r for the one-year bond, and $1.8r$ for the two-year bond. Determine r. [9.i-n #06]

[A] 8.98% B) 8.65% C) 9.31% D) 9.65% E) 9.98%



$$L = 1000 + 5680r$$
$$2L = 2600 + 4680r$$
$$\rightarrow 0 = 600 - 6680r$$
$$r = \boxed{8.982\%}$$

4. Dave needs to pay 1400 at the end of each year for the next 3 years. He purchased 3 different annual coupon bonds to match the liabilities exactly. The bonds have the following properties:

Bond	Time to Maturity	Coupon Rate
A	1	0.08
B	2	*X*
C	3	0.09

All three bonds have a par value of 1000 and can be redeemed at par. Dave purchased 1.16 units of Bond *A*. Find *X*, the coupon rate on bond *B*. [09i_A1]

A) 2.52% B) 2.43% C) 2.62% D) 2.71% E) 2.8%

$t=3$: $1400 = C(1.09)(1000)$

$t=2$: $1400 = C(0.09)(1000) + B(1+x)(1000)$

$t=1$: $1400 = C(0.09)(1000) + B(x)(1000) + (1.16)(1.08)(1000)$

$t=3$ Eqn $\Rightarrow$ $C = 1.2844$

$t=2$, $t=1$ $\Rightarrow$ $1.4 = 0.1156 + B(1+x)$

$1.4 = 0.1156 + Bx + 1.2528$

$\Rightarrow B = 1.2528$

$x = \boxed{2.523\%}$

5. A company has liabilities of 25,000 at the end of year 1, 35,000 at the end of year 2, and 55,000 at the end of year 3. The company plans to exactly match the liabilities by investing in the following bonds:

I. A one-year zero coupon bond with a yield of 3.5%
II. A two-year zero coupon bond with a yield of 4%
III. A three-year [redacted] bond paying annual coupons of 6% and yielding 5.5%

Find the total amount invested in the three bonds. Round your answer to the nearest 50. [09i_A4]

[A] 103,200 B) 104,750 C) 106,300 D) 107,850 E) 109,400

$t=3$: $55 = 1.06 F_3$ $\quad F_3 = 51.887$

$t=2$: $35 = 0.06 F_3 + F_2$ $\quad F_2 = 31.887$

$t=1$: $25 = 0.06 F_3 + F_1$ $\quad F_1 = 21.887$

$$PV = \frac{F_1}{1.035} + \frac{F_2}{(1.04)^2} + 3.1132\,a_{\overline{3}|5.5\%} + \frac{F_3}{(1.055)^3}$$

$$= \boxed{103.215}$$