## HW 2.2 Key

1. You are given that $P(A) = 0.35$ and $P(A \cup B) = 0.74$.

   Let $x = P(B)$, calculated under the ~~assume~~ assumption that $A$ and $B$ are independent.

   Let $y = P(B)$, calculated under the ~~assume~~ that $A$ and $B$ are mutually exclusive.

   Find $x - y$.

   A) 0.2100
   B) 0.1932
   C) 0.2016
   D) 0.2184
   E) 0.2268

In general: $0.74 = 0.35 + P(B) - P(A \cap B)$

$P(B) = 0.39 + P(A \cap B)$

Assume $A \perp B$: $P(B) = 0.39 + 0.35\,P(B)$

$P(B) = 0.6 = x$

Assume $A \cap B = \phi$: Then $P(A \cap B) = 0$

$P(B) = 0.39 = y$

$x - y = \boxed{0.21}$

2. It is known that 3.9% of the population has a certain disease. A new test is developed to screen for the disease. A study has shown that the test returns a positive result for 11% of all individuals, and returns a positive result for 80% of individuals who do have the disease.

   If a person tests positively for the disease under this test, what is the probability that they actually have the disease?

   A) 0.2836
   B) 0.2496
   C) 0.2609
   D) 0.2723
   E) 0.2950

$P(D) = 0.039$

$P(+) = 0.11$

$P(+|D) = 0.8$

$P(D \cap +) = P(+|D)\,P(D) = 0.0312$

$P(D|+) = \dfrac{P(D \cap +)}{P(+)} = \boxed{0.2836}$

3. While studying a certain population, researchers determine that 41% of individuals within the population have high blood pressure, and 14% are smokers. Of those who have high blood pressure, 24% are smokers. What is the probability that a smoker will have high blood pressure?

A) 0.7029
B) 0.5904
C) 0.6185
D) 0.6466
E) 0.6747

$P(H) = 0.41$

$P(S) = 0.14$

$P(S|H) = 0.24$

$P(S \cap H) = P(S|H)\,P(H) = 0.0984$

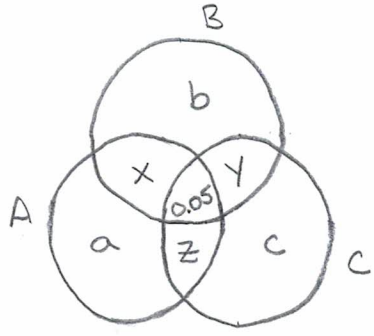
$P(H|S) = \dfrac{P(H \cap S)}{P(S)} = \boxed{0.7029}$

4. You are given the following information about events $A$, $B$, and $C$.
   - $P(A) = 0.4,\ P(B) = 0.3,\ P(C) = 0.47$
   - Events $A$ and $B$ are independent.
   - $P(C|A) = 1.5P(C|B)$
   - The probability of at least two of these events occurring is 0.27.
   - The probability of exactly two of these events occurring is 0.22.

$\rightarrow P(A \cap B \cap C) = 0.05$

Find $P(A|C)$.

A) 0.3546
B) 0.3440
C) 0.3652
D) 0.3759
E) 0.3865




$P(C|A) = 1.5\,P(C|B) \Rightarrow \dfrac{P(A \cap C)}{P(A)} = 1.5\,\dfrac{P(B \cap C)}{P(B)} \Rightarrow P(A \cap C) = 2\,P(B \cap C)$

$\Rightarrow 0.05 + z = 2(0.05 + y) \Rightarrow 2y - z = -0.05$

$A \perp B \Rightarrow P(A \cap B) = P(A)\,P(B) \Rightarrow x + 0.05 = 0.12 \Rightarrow x = 0.07$

$x + y + z = 0.22 \rightarrow \begin{matrix} y + z = 0.15 \\ 2y - z = -0.05 \end{matrix}\Big\} \begin{matrix} z = 0.03333 \\ y = 0.11667 \end{matrix}$

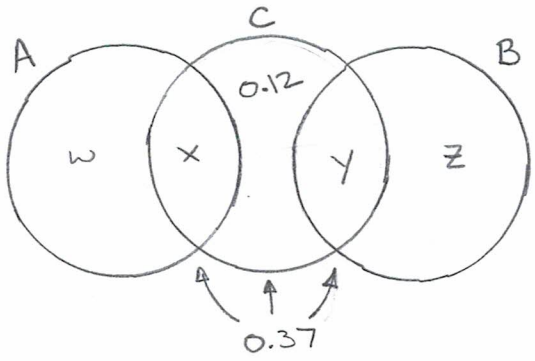
$P(A|C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{0.16667}{0.47} = \boxed{0.3546}$

5. A small online retailer sells three products: Product A, Product B, and Product C. A study of the purchasing history of their customers reveals the following:

1 • No customers purchased both Product A and Product B.
2 • 1.4 times as many customers purchased Product A as Product B.
3 • 37% of customers purchased Product C.
4 • 12% of customers purchased only Product C.
5 • The event that a customer purchased A is independent of the event that they purchased C.
6 • The event that a customer purchased B is independent of the event that they purchased C.

Calculate the probability that a randomly selected customer purchased exactly one of the products.

A) 0.5457
B) 0.5784
C) 0.6112
D) 0.6439
E) 0.6766




② $w + x = 1.4(y + z) \rightarrow w + x - 1.4y - 1.4z = 0$

③+④ $x + y = 0.25 \rightarrow y = 0.25 - x$

⑤ $x = 0.37(w + x) \rightarrow x = 0.5873w$

⑥ $y = 0.37(y + z) \rightarrow z = 1.7027y$

$w + x - 1.4y - 1.4(1.7027)y = 0$
$w + x - 3.7838y = 0$
$w + x - 3.7838(0.25 - x) = 0$
$w + 4.7838x = 0.9459$
$w + 4.7838(0.5873)w = 0.9459$
$3.8095w = 0.9459$

$w = 0.2483$
$x = 0.1458$
$y = 0.1042$
$z = 0.1774$

$w + 0.12 + z = \boxed{0.5457}$