HW 2.3 Key

1. You are given the following information regarding a new test for a certain medical condition:
   - For individuals who have the condition, the probability that the test returns a positive result is 0.84.
   - For individuals who do not have the condition, the probability that the test returns a positive result is 0.11.
   - The proportion of individuals in the population who have the condition is 0.09.

   Assume that a person is tested, yielding a positive result. Determine the probability that the individual does, in fact, have the condition.

   A) 0.4303
   B) 0.3786
   C) 0.3916
   D) 0.4045
   E) 0.4174

$P(+|C) = 0.84 \qquad P(+|C') = 0.11 \qquad P(C) = 0.09$

$$P(C|+) = \frac{P(C \cap +)}{P(+)} = \frac{P(C \cap +)}{P(+ \cap C) + P(+ \cap C')}$$

$$= \frac{P(+|C)P(C)}{P(+|C)P(C) + P(+|C')P(C')}$$

$$= \frac{0.84(0.09)}{0.84(0.09) + 0.11(0.91)} = \boxed{0.4303}$$

2. You are given the following information regarding a new test for a certain medical condition:
   - For individuals who have the condition, the probability that the test returns a positive result is 0.8.
   - For individuals who do not have the condition, the probability that the test returns a positive result is 0.23.
   - The proportion of individuals in the population who have the condition is 0.08.

   Assume that a person is tested, yielding a negative result. Determine the probability that the individual does not have the condition.

   A) 0.9779
   B) 0.9584
   C) 0.9632
   D) 0.9681
   E) 0.9730

$P(+|C) = 0.8 \qquad P(+|C') = 0.23 \qquad P(C) = 0.08$

$$P(C'|-) = \frac{P(C' \cap -)}{P(-)} = \frac{P(- \cap C')}{P(- \cap C') + P(- \cap C)}$$

$$= \frac{P(-|C')P(C')}{P(-|C')P(C') + P(-|C)P(C)}$$

$$= \frac{0.77(0.92)}{0.77(0.92) + 0.2(0.08)} = \boxed{0.9779}$$

3. A student takes a test consisting of 25 true-false questions. The student knows for certain the correct answer to K questions, and guesses on the remaining questions. For any question that the student ~~answer~~ answered correctly, the probability that they guessed on it is 0.3889. Find K.

A) 11
B) 9
C) 10
D) 12
E) 13

KA = Knew Answer C = Got Question Correct

$P(C|KA) = 1 \qquad P(C|KA') = 0.5$

$P(KA) = \frac{K}{25}$

$$P(KA'|C) = 0.3889 = \frac{P(KA' \cap C)}{P(C)} = \frac{P(C \cap KA')}{P(C \cap KA') + P(C \cap KA)}$$

$$0.3889 = \frac{0.5\left(\frac{25-K}{25}\right)}{0.5\left(\frac{25-K}{25}\right) + 1\left(\frac{K}{25}\right)} = \frac{12.5 - 0.5K}{12.5 + 0.5K}$$

$$0.3889(12.5 + 0.5K) = 12.5 - 0.5K \Rightarrow K = \boxed{11}$$

4. A lending agency classifies each of its clients into one of three risk categories: low risk, medium risk, or high risk. 24% of the clients are low risk, 32% are medium risk, and 44% are high risk. The probability that a high risk client defaults on their loan is 2.1 times that of a medium risk client. The probability that a medium risk client defaults on their loan is 2.4 times that of a low risk client.

Given that a client defaulted on their loan, determine the probability that they were high risk.

A) 0.6875
B) 0.7081
C) 0.7288
D) 0.7494
E) 0.7700

$P(L) = 0.24 \qquad P(M) = 0.32 \qquad P(H) = 0.44$

$P(D|H) = 2.1\,P(D|M) \qquad P(D|M) = 2.4\,P(D|L)$

$$P(H|D) = \frac{P(D|H)\,P(H)}{P(D|H)\,P(H) + \frac{1}{2.1}P(D|H)\,P(M) + \frac{1}{2.1(2.4)}P(D|H)\,P(L)}$$

$$= \frac{0.44}{0.44 + \frac{0.32}{2.1} + \frac{0.24}{2.1(2.4)}}$$

$$= \boxed{0.6875}$$

5. Professor Allen and Professor Brown each teach sections of the same statistics course. Professor Allen has 53 students, and Professor Brown has 47 students. On any given day, the probability that Professor Allen will give a pop quiz is 0.28 and the probability that Professor Brown will give a pop quiz is 0.22. It is known that 30% of students will fail a pop quiz in Professor Allen's class, and 18% of students will fail a pop quiz in Professor Brown's class.

   Assume that a student in one of the classes has just failed a pop quiz. Determine the probability that the student was in Professor Allen's class.

   A) 0.7052
   B) 0.6417
   C) 0.6629
   D) 0.6840
   E) 0.7263

$P(A) = 0.53$ $\quad P(B) = 0.47$

$P(Q|A) = 0.28$ $\quad P(Q|B) = 0.22$

$P(F|Q \cap A) = 0.30$ $\quad P(F|Q \cap B) = 0.18$

$$P(A|Q \cap F) = \frac{P(A \cap Q \cap F)}{P(Q \cap F)} = \frac{P(Q \cap A \cap F)}{P(Q \cap A \cap F) + P(Q \cap B \cap F)}$$

$$= \frac{P(F|Q \cap A)\, P(Q|A)\, P(A)}{P(F|Q \cap A) P(Q|A) P(A) + P(F|Q \cap B) P(Q|B) P(B)}$$

$$= \frac{0.30(0.28)(0.53)}{0.30(0.28)(0.53) + 0.18(0.22)(0.47)}$$

$$= \boxed{0.7052}$$