HW 2.4 Key

1. A college club consists of 49 members. Of the members, 14 are seniors, 16 are juniors, 13 are sophomores, and 6 are freshmen. The club needs to elect an executive board consisting of a President, Vice President, a Treasurer, and a Secretary. The club's constitution requires that the President is a senior and Vice President is a junior. A student is not allowed to hold more than one office. Determine the number of possible ways in which the board could be formed.

   A) 484,288
   B) 474,602
   C) 493,974
   D) 503,660
   E) 513,345

   $$14(16)(47)(46) = 484,288$$

2. The board of directors for a particular company consists of 7 women and 20 men. A committee of 5 people is selected at random from the members of the board. Find the probability that the committee consists only of men.

   A) 0.1920
   B) 0.1978
   C) 0.2036
   D) 0.2093
   E) 0.2151

   $$P(\text{All men}) = \frac{_{20}C_5}{_{27}C_5} = \frac{15,504}{80,730}$$

   $$= 0.1920$$

3. The board of directors for a particular company consists of 6 women and 18 men. A committee of 5 people is selected at random from the members of the board. Find the probability that the committee consists ~~only~~ of 3 women and 2 men.

A) 0.0720
B) 0.0677
C) 0.0698
D) 0.0742
E) 0.0763

$$P(3 \text{ women}, 2 \text{ men}) = \frac{(6C_3)(18C_2)}{24C_5}$$

$$= \frac{20(153)}{42,504}$$

$$= \boxed{0.0720}$$

4. A contracting agency has three clients that need to hire contract workers to fill various positions. Client A requires 5 workers, Client B requires 2 workers, and Client C requires 6 workers. The agency has 17 employees that can be assigned to these clients. No employee can be assigned to more than one client. Determine the number of ways in which the agency can assign its available employees to the three clients.

A) 85,765,680
B) 80,619,739
C) 83,192,710
D) 88,338,650
E) 90,911,621

$$\frac{17!}{5! \, 2! \, 6! \, 4!} = \boxed{85,765,680}$$

5. A class of 40 students is to be divided into three groups with sizes 14, 12, and 14. Alice and Beth are best friends, and hope to be in the same group. If the groups are assigned randomly, what is the probability that Alice and Beth will be in the same group?

A) 0.3179
B) 0.2989
C) 0.3084
D) 0.3275
E) 0.337

# of possible assignments $= \dfrac{40!}{14!\,12!\,14!} = D$

# of assignments in which Alice and Beth are in the same group =

$$= \frac{38!}{12!\,12!\,14!} + \frac{38!}{14!\,10!\,14!} + \frac{38!}{14!\,12!\,12!} = N$$

$$\frac{N}{D} = 38!\left(\frac{1}{12!\,12!\,14!} + \frac{1}{14!\,10!\,14!} + \frac{1}{14!\,12!\,12!}\right)\frac{14!\,12!\,14!}{40!}$$

$$= \frac{38!}{40!}\left(\frac{14!\,12!\,14!}{12!\,12!\,14!} + \frac{14!\,12!\,14!}{14!\,10!\,14!} + \frac{14!\,12!\,14!}{14!\,12!\,12!}\right)$$

$$= \frac{1}{40(39)}\left[14(13) + 12(11) + 14(13)\right] = \boxed{0.3179}$$

Or:

$$P(A \text{ in } G_1)P(B \text{ in } G_1 | A \text{ in } G_1) + P(A \text{ in } G_2)P(B \text{ in } G_2 | A \text{ in } G_2) + P(A \text{ in } G_3)P(B \text{ in } G_3 | A \text{ in } G_3)$$

$$= \frac{14}{40}\cdot\frac{13}{39} + \frac{12}{40}\cdot\frac{11}{39} + \frac{14}{40}\cdot\frac{13}{39} = \boxed{0.3179}$$