HW 3.2 Key

1. Assume that X is a discrete random variable that can take on the following integer values: $x = 1, 2, 3, 4, 5, 6$. The distribution satisfies the property that $P(X = x+1) = 1.25P(X = x)$ for all $x \le 5$. Find $P(X = 5)$.

A) 0.2168 B) 0.2871 C) 0.3079 D) 0.3183 E) 0.3287

Let $f(1) = k$. Then $f(x) = k(1.25)^{x-1}$.

$$k\left[1 + 1.25 + 1.25^2 + 1.25^3 + 1.25^4 + 1.25^5\right] = 1$$

$$k\left[\frac{1 - 1.25^6}{1 - 1.25}\right] = 1 \Rightarrow k = 0.088819$$

$$f(5) = 0.088819(1.25)^4 = \boxed{0.2168}$$

2. Let X be a discrete random variable with probability mass function given by $f(x) = k(0.7)^x$ for $x = 0, 1, 2, 3, \ldots$. Find $P(X = 3)$.

A) 0.1029 B) 0.1065 C) 0.1101 D) 0.1137 E) 0.1173

$$\sum_{x=0}^{\infty} k(0.7)^x = 1$$

$$k\left[1 + 0.7 + 0.7^2 + \ldots\right] = k\,\frac{1}{1-0.7} = 1 \Rightarrow k = 0.3$$

$$f(3) = 0.3(0.7)^3 = \boxed{0.1029}$$

3. Let X be a discrete random variable that can take on values $x = 1, 2, \ldots, 8$. Assume that the probability mass function for X satisfies the expression $f(x) = f(x-1) + 0.01$ for all $x > 1$. Find $P(x > 3)$.

A) 0.7000 B) 0.7250 C) 0.7533 D) 0.7816 E) 0.8383

Let $f(1) = k$. Then $f(x) = k + (x-1)(0.01)$

$f(1) + f(2) + \ldots + f(8) = [k + (k+0.01) + (k+2(0.01)) + \ldots + (k+7(0.01))]$

$= 8k + 0.01(1+2+3+\ldots+7) = 8k + 0.01\,\frac{7 \cdot 8}{2}$

$= 8k + 0.01(28) = 1 \Rightarrow k = 0.09$

$P[X > 3] = 1 - f(1) - f(2) - f(3)$

$= 1 - [0.09 + 0.10 + 0.11] = \boxed{0.7}$

4. Let X be a discrete random variable with probability mass function given by the following table:

x	1	2	3	4	5	6	7	8
$p(x)$	0.24	0.21	0.17	0.12	0.10	0.09	0.05	0.02

Find the probability that X is at least 5, given that it is greater than 2.

A) 0.4727 B) 0.4231 C) 0.4396 D) 0.4562 E) 0.4893

$P[X \geq 5 \mid X > 2] = \dfrac{P[X \geq 5 \cap X > 2]}{P[X > 2]}$

$= \dfrac{P[X \geq 5]}{1 - P[X \leq 2]}$

$= \dfrac{0.26}{1 - 0.45}$

$= \boxed{0.4727}$

5. Let X be a discrete random variable with probability mass function given by $f(x) = k(0.63)^x$ for $x = 0, 1, 2, 3, \ldots$. Find $P(X \geq 5 | X \geq 3)$.

A) 0.3969 B) 0.3552 C) 0.3691 D) 0.3830 E) 0.4108

$$\sum_{x=0}^{\infty} k(0.63)^x = k \frac{1}{1-0.63} = 1 \Rightarrow k = 0.37$$

$$P[X \geq 5 \mid X \geq 3] = \frac{P[X \geq 5 \cap X \geq 3]}{P[X \geq 3]}$$

$$= \frac{P[X \geq 5]}{P[X \geq 3]}$$

$$= \frac{1 - 0.37(1 + 0.63 + 0.63^2 + 0.63^3 + 0.63^4)}{1 - 0.37(1 + 0.63 + 0.63^2)}$$

$$= \frac{0.099244}{0.250047}$$

$$= \boxed{0.3969}$$