

HW 3.3(a) Key

1. Let X be a discrete random variable with probability mass function given by the following table:

x	50	100	150	200	250	300
$p(x)$	0.26	0.21	0.17	0.14	0.12	0.10

Find the variance of X .

- A) 6918.75 B) 6489.79 C) 6704.27 D) 7133.23 E) 7347.71

$$E[X] = 0.26(50) + 0.21(100) + 0.17(150) + 0.14(200) + 0.12(250) + 0.1(300) \\ = 147.50$$

$$E[X^2] = 0.26(50)^2 + 0.21(100)^2 + 0.17(150)^2 + 0.14(200)^2 + 0.12(250)^2 + 0.1(300)^2 \\ = 28,675$$

$$\text{Var}[X] = 28,675 - (147.50)^2 \\ = \boxed{6918.75}$$

2. Let X be a discrete random variable with probability mass function given by $f(x) = 0.038 + 0.054x$ for $x = 1, 2, 3, 4, 5$. Find the standard deviation of X .

- A) 1.3071 B) 1.1764 C) 1.2417 D) 1.3724 E) 1.4378

$$E[X] = 0.092(1) + 0.146(2) + 0.2(3) + 0.254(4) + 0.308(5) \\ = 3.54$$

$$E[X^2] = 0.92(1) + 0.146(4) + 0.2(9) + 0.254(16) + 0.308(25) \\ = 14.24$$

$$\text{Var}[X] = 14.24 - (3.54)^2 = 1.7084$$

$$\text{SD}[X] = \boxed{1.3071}$$

3. Let X be a discrete random variable with probability mass function given by the table below.

x	1	2	3	4	5	6	7	8
$p(x)$	0.020	0.050	0.080	0.110	0.140	0.170	0.200	0.230

Calculate the probability that a randomly selected observation of X falls within one standard deviation of the mean.

- A) 0.620 B) 0.496 C) 0.527 D) 0.558 E) 0.589

$$E[X] = \sum_{x=1}^8 x f(x) = 5.76 \quad E[X^2] = \sum_{x=1}^8 x^2 f(x) = 36.84$$

$$\text{Var}[X] = 36.84 - (5.76)^2 = 3.6624$$

$$\text{SD}[X] = 1.9137$$

$$P[3.8463 \leq X \leq 7.6737] = 0.110 + 0.140 + 0.170 + 0.200$$

$$= \boxed{0.620}$$

4. When purchasing a new smart phone, Anna also purchases a warranty on the phone. The warranty will pay Anna 240 if the phone fails within the first year after the purchase, 180 if it fails during the second year, 120 if it fails during the third year, and 60 if it fails during the fourth year. If the phone fails after four years, then the warranty pays nothing. In any given year, there is a 7% chance that the phone fails during that year, assuming it did not fail during any of the previous years. Calculate the expected amount that the warranty will pay to Anna.

- A) 39.16 B) 36.26 C) 37.71 D) 40.61 E) 42.06

X = year in which the phone fails.

$$f(x) = (0.93)^{x-1} (0.07), \quad x = 1, 2, 3, \dots$$

W = Warranty Payment

$$E[W] = 240 f(1) + 180 f(2) + 120 f(3) + 60 f(4)$$

$$= 240 (0.07) + 180 (0.0651) + 120 (0.060543) + 60 (0.05630499)$$

$$= \boxed{39.16}$$

5. Steven purchases a special type of financial instrument, called an option. The option will potentially make a one time payment to Steven one year from now, depending on the value of a certain stock at that time. Let S be a random variable that represents the price of the stock one year from now.

Steven's option is set up so that it will pay $\max(S - 205, 0)$ one year from now. In other words, if $S > 205$, then the option will pay an amount of $S - 205$ to Steven. If $S \leq 205$, then Steven will receive no money from the option.

Assume that S follows the following discrete distribution:

S	160	180	200	220	240
$p(S)$	0.08	0.23	0.38	0.23	0.08

Find the variance of the amount paid to Steven by the option.

- A) 110.69 B) 116.22 C) 121.76 D) 127.29 E) 132.82

$P = \text{Option Payment}$

P	0	15	35
$f(p)$	0.69	0.23	0.08

$$E[P] = 0.69(0) + 0.23(15) + 0.08(35) = 6.25$$

$$E[P^2] = 0.69(0) + 0.23(15)^2 + 0.08(35)^2 = 149.75$$

$$\text{Var}[P] = 149.75 - (6.25)^2 = \boxed{110.6875}$$