HW 3.3(b) Key

1. Let X be a discrete random variable such that $E[X]=5.9$ and $Var[X]=4.8$. Let $Y=5X+8$. Find $E[Y^2]$.

A) 1526.25 B) 1373.62 C) 1411.78 D) 1449.94 E) 1488.09

$Var[X] = E[X^2] - (E[X])^2 = E[X^2] - (5.9)^2 = 4.8 \Rightarrow E[X^2] = 39.61$

$E[Y^2] = E[25X^2 + 80X + 64] = 25E[X^2] + 80E[X] + 64$

$= 25(39.61) + 80(5.9) + 64 = \boxed{1526.25}$

2. Let X be a discrete random variable such that $E[X]=3.7$, $E[X^3]=51.5$, and $Var[X]=1.4$. Find $E[(x-1)^3]$.

A) 16.33 B) 14.37 C) 15.02 D) 15.68 E) 16.98

$Var[X] = E[X^2] - (3.7)^2 = 1.4 \Rightarrow E[X^2] = 15.09$

$E[(X-1)^3] = E[X^3 - 3X^2 + 3X - 1] = E[X^3] - 3E[X^2] + 3E[X] - 1$

$= 51.5 - 3(15.09) + 3(3.7) - 1$

$= \boxed{16.33}$

3. Let X be a discrete random variable, and let $Z=3X+12$. If $E[Z]=96$ and $Var[Z]=54$, find $E[X^2]$.

A) 790 B) 758 C) 774 D) 806 E) 822

$E[Z] = 3E[X] + 12 = 96 \Rightarrow E[X] = 28$

$Var[Z] = Var[3X+12] = 9Var[X] = 54 \Rightarrow Var[X] = 6$

$E[X^2] = Var[X] + (E[X])^2 = 6 + 28^2 = \boxed{790}$

4. Let X be a discrete random variable. Let $Y = 6X + 20$ and $Z = 5X^2 + 2X + 13$. Given that $E[Y] = 206$ and $E[Z] = 4940$, find $Var[X]$.

[A] 12 B) 11 C) 13 D) 14 E) 15

$E[Y] = 6E[X] + 20 = 206 \Rightarrow E[X] = 31$

$E[Z] = 5E[X^2] + 2E[X] + 13 = 4940 \Rightarrow E[X^2] = 973$

$Var[X] = 973 - (31)^2 = \boxed{12}$

5. Let X be a discrete random variable such that $Var[X] = 82.35$. Let $Y = X^2 - 15.3X + 8$. Given that $E[Y] = 32.93$ and $E[X^2] < 140$, find $E[X]$.

[A] 6.6 B) 6.3 C) 9.1 D) 8.7 E) 7.6

$Var[X] = E[X^2] - \mu^2 = 82.35$

$E[Y] = E[X^2] - 15.3\mu + 8 = 32.93$

$E[X^2] - \mu^2 = 82.35$

$E[X^2] - 15.3\mu = 24.93$

$\downarrow$

$\mu^2 - 15.3\mu + 57.42 = 0 \Rightarrow \mu = 8.7$ or $\mu = 6.6$

$\mu = 8.7 \Rightarrow E[X^2] = 158.04$

$\mu = 6.6 \Rightarrow E[X^2] = 125.91$ $\boxed{\mu = 6.6}$