HW 3.4 Key

1. Bridgett is taking a multiple choice final exam with 30 questions. Each question has 5 options for the answer. Bridgett is certain that she knows the correct answer for 16 of the problems, and guesses randomly on the remaining 14 questions. Bridgett needs to correctly answer at least [redacted] 20 questions on the final to pass the class. Assuming that she correctly answers the 16 questions that she did not guess on, what is the probability that Bridgett will pass the class?

   $\boxed{\text{A)}}$ 0.3018 B) 0.3139 C) 0.3260 D) 0.3380 E) 0.3501

   Let X = Number of correct guesses $\qquad f(x) = \binom{14}{x}(0.2)^x(0.8)^{14-x}$

   $P[X \geq 4] = 1 - [f(0) + f(1) + f(2) + f(3)]$

   $= 1 - [0.8^{14} + 14(0.8)^{13}(0.2) + 91(0.8)^{12}(0.2)^2 + 364(0.8)^{11}(0.2)^3]$

   $= 1 - 0.6982 = \boxed{0.3018}$

2. The probability that it rains during any given day during the next 7 days is equal to *p*. The events that it rains on any two given days are independent of each other. The probability that it rains on 2 or fewer of the next 7 days is 3.482 times the probability that it rains on more than 2 of the next 7 days. Find *p*.

   $\boxed{\text{A)}}$ 0.24 B) 0.23 C) 0.25 D) 0.26 E) 0.28

   Let X = Number of days with rain $\qquad f(x) = \binom{7}{x} p^x q^{7-x}$

   Let $k = P[X \leq 2]$. Then $k = 3.482(1-k) \Rightarrow k = \frac{3.482}{4.482} = 0.7769$

   $f(0) + f(1) + f(2) = 0.7769$

   $(1-p)^7 + 7p(1-p)^6 + 21p^2(1-p)^5 = 0.7769$

   Using "table" function on the TI-30XS:

   $p = \boxed{0.24}$

3. Gloria plays a game that involves repeatedly rolling a 20-sided die. The sides of the die are numbered 1 through 20, and each result is equally likely to appear as the result of the roll. Gloria rolls the die 11 times and is awarded 3 points for each roll that results in value less than or equal to 7. Let *T* be the total number of points scored by Gloria. Find $Var[T]$.

   $\boxed{\text{A)}}$ 22.52 B) 21.62 C) 23.42 D) 24.32 E) 25.23

   X = number of rolls ≤ 7 $\qquad n = 11 \qquad p = 0.35 \qquad q = 0.65$

   $T = 3X$

   $Var[T] = 9\,Var[X] = 9npq = 9(11)(0.35)(0.65) = \boxed{22.5225}$

4. Felix pays 280 to play an electronic game of chance at a casino. The rules of the game are as follows: The game will randomly [redacted] simulate a sequence of 30 coin flips. The probability that any given flip comes up as heads is equal to *p*, and is unknown to Felix. After all 30 flips are generated, the game will pay Felix an amount equal to X^2, were *X* is the number of flips resulting in heads. Determine the smallest value of *p* for which Felix's expected winnings are greater than zero.

$\boxed{\text{A)}}$ 0.5503 B) 0.4623 C) 0.4843 D) 0.5063 E) 0.5283

X = number of heads $n = 30$

W = net winnings

$E[X] = 30p$ $Var[X] = 30p(1-p)$

$E[X^2] = Var[X] + (E[X])^2 = 30p - 30p^2 + 900p^2 = 870p^2 + 30p$

$E[W] = E[X^2 - 280] = 870p^2 + 30p - 280$

$870p^2 + 30p - 280 = 0 \Rightarrow p = 0.5503$ or $p = -0.5848$

$E[W] > 0$ if $p > \boxed{0.5503}$

5. A company offers guided tours of remote areas of the world. Each tour consists of 20 people. The probability of selling a ticket for any given spot on the tour is [redacted] 0.87. The costs to the company are approximately the same regardless of the number of people that show up. To offset the risk associated with a tour not filling up, the company purchases a specialized insurance policy. The policy pays 60 for each ticket that is left unsold, up to a maximum of 240. Let *P* be the amount that the insurance policy pays to the company for a specific tour. Calculate $E[P]$ to the nearest dollar.

$\boxed{\text{A)}}$ 146 B) 138 C) 142 D) 151 E) 155

Let X = Number of unsold tickets. $f(x) = \binom{20}{x}(0.13)^x(0.87)^{20-x}$

$$P = \begin{cases} 60x & \text{if } x \le 3 \\ 240 & \text{if } x \ge 4 \end{cases}$$

$$E[P] = 0 \cdot f(0) + 1 \cdot f(1) + 2 \cdot f(2) + 3 \cdot f(3) + 240\left[1 - \left(f(0) + f(1) + f(2) + f(3)\right)\right]$$

$$= 0(0.0617) + 60(0.1844) + 120(0.2618) + 180(0.2347) + 240(0.2574)$$

$$= \boxed{146.502}$$