HW 3.5 Key

1. A biased coin is weighted so that the probability that a flip of the coin results in heads is equal to 0.61. The coin is flipped repeatedly. Find the probability that the first flip resulting in tails occurred within the first 4 flips.

   $\boxed{\text{A)}}$ 0.8615 B) 0.9046 C) 0.9477 D) 0.9908 E) 1.0338

   X = # of flips req'd to get 1st Tails

   $X \sim GEOM(p = 0.39) \qquad f(x) = (0.61)^{x-1}(0.39)$

   $P[X \le 4] = f(1) + f(2) + f(3) + f(4) = 0.39[1 + 0.61 + 0.61^2 + 0.61^3]$

   $= \boxed{0.8615}$

2. A casino gamer is playing a slot machine. The chance ~~the~~ that any given play on the slot machine results in a win is constant, and is independent from the results of any other plays on the machine. The expected number of times that the gamer will need to play the slot machine to get her first win is equal to 13. Find the probability that the gamer gets a win in 8 or fewer plays of the machine.

   A) 0.4729 B) 0.4965 C) 0.5202 D) 0.5438 E) 0.5675

   X = # of plays req'd to get 1st Win

   $X \sim GEOM(p) \qquad E[X] = 13 \Rightarrow \frac{1}{p} = 13 \Rightarrow p = \frac{1}{13}$

   $P[X \le 8] = f(1) + f(2) + \dots + f(8) = \frac{1}{13}\left[1 + \frac{12}{13} + \left(\frac{12}{13}\right)^2 + \dots + \left(\frac{12}{13}\right)^7\right]$

   $= \frac{1}{13} \frac{1 - \left(\frac{12}{13}\right)^8}{1 - \frac{12}{13}} = \frac{1 - \left(\frac{12}{13}\right)^8}{13 - 12} = 1 - \left(\frac{12}{13}\right)^8 = \boxed{0.4729}$

3. Darren is a contestant on a game show. He is presented with a game that has two outcomes: win or lose. Darren is asked to play the game repeatedly until losing. He will win $100X^2$, where X is the number of games that he wins before getting his first loss. The probability of winning on any given attempt at the game is 0.46, and is independent from each other attempt. Find the expected value of Darren's winnings, rounded to the nearest whole number.

   A) 230 B) 207 C) 219 D) 242 E) 253

   Y = # of plays req'd to get 1st loss

   $Y \sim GEOM(p = 0.54)$

   $E[Y] = \frac{1}{p} = 1.85185 \qquad Var[Y] = \frac{q}{p^2} = 1.57750 \qquad E[Y^2] = 5.00686$

   $X = Y - 1$

   $E[100X^2] = 100E[X^2] = 100E[(Y-1)^2] = 100E[Y^2 - 2Y + 1]$

   $= 100(E[Y^2] - 2E[Y] + 1) = \boxed{230.32}$

4. Let X be a discrete random variable following a geometric distribution with $p = 0.13$. Let Y be another discrete random variable defined by $Y = \min(0, X-4)$. In other words, $Y = 0$ if $X \le 4$, and $Y = X - 4$ if $X > 4$. Find $Var[Y]$.

A) 43.9708 B) 46.1694 C) 48.3679 D) 50.5664 E) 52.7650

$$E[Y] = 1\cdot f(5) + 2\cdot f(6) + 3\cdot f(7) + 4\cdot f(8) + \dots$$
$$= 1q^4p + 2q^5p + 3q^6p + 4q^7p + \dots$$
$$= q^4[1\cdot p + 2qp + 3q^2p + 4q^3p + \dots] = q^4 E[X] = \frac{q^4}{p}$$

$$E[Y^2] = 1^2\cdot f(5) + 2^2\cdot f(6) + 3^2\cdot f(7) + 4^2 f(8) + \dots$$
$$= 1^2 q^4 p + 2^2 q^5 p + 3^2 q^6 p + 4^2 q^7 p + \dots$$
$$= q^4[1^2 p + 2^2 qp + 3^2 q^2 p + 4^2 q^3 p + \dots] = q^4 E[X^2]$$

$$E[X^2] = Var[X] + (E[X])^2 = \frac{q}{p^2} + \frac{1}{p^2}$$

$$Var[Y] = E[Y^2] - (E[Y])^2 = q^4\left(\frac{q}{p^2} + \frac{1}{p^2}\right) - \left(\frac{q^4}{p}\right)^2 = \boxed{43.9708}$$

5. Megan and Rick are baseball pitchers. They are practicing by throwing baseball at a small target. They each throw balls at the target until they miss the target for the first time. Megan has a 65% chance of hitting the target on any given throw. Rick has a 57% chance of hitting the target on any given throw. Find the probability that Megan hits the target more times than Rick.

A) 0.4440 B) 0.5560 C) 0.3169 D) 0.6831 E) 0.2391

M = # throws req'd for Megan to miss $\quad M \sim GEOM(p=0.35)$

R = # throws req'd for Rick to miss $\quad R \sim GEOM(p=0.43)$

$$P[R<M] = P[R=1, M>1] + P[R=2, M>2] + P[R=3, M>3] + \dots$$
$$= (0.43)(0.65) + [(0.43)(0.57)](0.65)^2 + [(0.43)(0.57)^2](0.65)^3 + \dots$$
$$= \frac{0.43(0.65)}{1 - 0.57(0.65)}$$
$$= \boxed{0.4440}$$

Note: We use the following fact in our calculation above:

$$P[M>k] = P[\text{First } k \text{ throws hit target}] = (0.65)^k$$