HW 3.7 Key

1. A manufacturer produces a batch of 90 widgets, 13 of which are defective. A sample of 6 widgets is drawn from the batch for testing. Find the probability that the number of defective widgets in the sample is equal to 2.

$\boxed{\text{A)}}$ 0.1695 B) 0.1492 C) 0.1543 D) 0.1594 E) 0.1644

X = # of defective widgets $\qquad X \sim HYP(N=90,\ r=13,\ n=6)$

$$P[X=2] = \frac{\binom{13}{2}\binom{77}{4}}{\binom{90}{6}} = \frac{78(1353275)}{622614630} = \boxed{0.1695}$$

2. A manufacturer produces a batch of 60 widgets, 20 of which are defective. A sample of 10 widgets is drawn from the batch for testing. Find the probability that the number of defective widgets in the sample less than the expected number of defective widgets to be found in the sample.

$\boxed{\text{A)}}$ ~~0.6582~~ 0.5595 B) 0.6812 C) 0.7042 D) 0.7273 E) 0.7503

X = # of defective widgets $\qquad X \sim HYP(N=60,\ r=20,\ n=10)$

$$E[X] = n\left(\frac{r}{N}\right) = 10 \cdot \frac{20}{60} = 3.33$$

$$P[X < 3.33] = f(0) + f(1) + f(2) + f(3)$$

$$= \frac{\binom{20}{0}\binom{40}{10}}{\binom{60}{10}} + \frac{\binom{20}{1}\binom{40}{9}}{\binom{60}{10}} + \frac{\binom{20}{2}\binom{40}{8}}{\binom{60}{10}} + \frac{\binom{20}{3}\binom{40}{7}}{\binom{60}{10}} = \boxed{0.5595}$$

3. A committee consists of 15 women and 12 men. A task force of 5 people is selected at random from the committee. Find the probability that the task force will contain more women than men.

A) 0.6121 B) 0.5937 C) 0.6304 D) 0.6488 E) 0.6672

$$P[\#\text{Women} > \#\text{Men}] = P[3W, 2M] + P[4W, 1M] + P[5W, 0M]$$

$$= \frac{\binom{15}{3}\binom{12}{2}}{\binom{27}{5}} + \frac{\binom{15}{4}\binom{12}{1}}{\binom{27}{5}} + \frac{\binom{15}{5}\binom{12}{0}}{\binom{27}{5}}$$

$$= \boxed{0.6121}$$

4. Abby is playing a lottery in which she is asked to select 5 distinct numbers from the integers 1 through 42. On the day of the lottery drawing, 5 numbers are drawn from the integers 1 through 42. Abby's winnings are equal to 10 times the square of the number of matches between her choices and the numbers drawn. Find Abby's expected winnings.

A) 8.28 B) 8.69 C) 9.10 D) 9.52 E) 9.93

X = # of matching numbers $\qquad X \sim HYP(N=42, r=5, n=5)$

$E[X] = 5\left(\frac{5}{42}\right) = 0.59524 \qquad Var[X] = 5\left(\frac{5}{42}\right)\left(\frac{37}{42}\right)\left(\frac{37}{41}\right) = 0.47322$

$E[X^2] = 0.82753$

W = Winnings $\qquad W = 10X^2$

$E[W] = 10\,E[X^2] = \boxed{8.2753}$

5. Cindy is playing a lottery in which she is asked to select 5 distinct numbers from the integers 1 through 31. On the day of the lottery drawing, 5 numbers are drawn from the integers 1 through 31. Cindy will win ~~100~~ 10 times the square of the number of matches between her choices and the numbers drawn. She will win and additional amount of 13,000 if she matches all 5 numbers drawn. Find Cindy's expected winnings rounded to the nearest dollar.

$\boxed{\text{A)}}$ 12.44 B) 11.82 C) 13.06 D) 13.69 E) 14.31

X = # matching numbers $\qquad X \sim HYP(N=31, r=5, n=5)$

$E[X] = 5\left(\frac{5}{31}\right) = 0.80645 \qquad Var[X] = 5\left(\frac{5}{31}\right)\left(\frac{26}{31}\right)\left(\frac{26}{30}\right) = 0.58619$

$E[X^2] = 1.23656$

W = Winnings $\qquad W = \begin{cases} 10X^2 & \text{if } X \le 4 \\ 10X^2 + 13000 & \text{if } X = 5 \end{cases}$

$E[W] = 10(1)^2 f(1) + 10(2)^2 f(2) + 10(3)^2 f(3) + 10(4)^2 f(4) + 10(5)^2 f(5)$
$+ 13{,}000\, f(5)$

$= 10\,E[X^2] + 13000\, f(5)$

$= 10(1.23656) + 13000 \dfrac{\binom{5}{5}\binom{26}{0}}{\binom{31}{5}} = \boxed{12.44}$