HW 3.8 Key

1. The number earthquakes in a certain region follows a Poisson distribution with a mean of 9 earthquakes per year. Find the probability that the number of earthquakes in a given year is at least 6, but less than 10.

$\boxed{\text{A)}}$ 0.4717 B) 0.4953 C) 0.5189 D) 0.5425 E) 0.5661

$$X \sim \text{POI}(\lambda = 9)$$

$$f(6) + f(7) + f(8) + f(9) = \frac{e^{-9}9^6}{6!} + \frac{e^{-9}9^7}{7!} + \frac{e^{-9}9^8}{8!} + \frac{e^{-9}9^9}{9!}$$

$$= \boxed{0.4717}$$

2. The number auto accidents on a certain ~~straight~~ section of highway follows a Poisson distribution with a mean of 8 accidents per month. Find the probability that the number of accidents in a certain month is at most 3, given that it is at most 5.

$\boxed{\text{A)}}$ 0.2216 B) 0.1773 C) 0.1884 D) 0.1995 E) 0.2105

$$X \sim \text{POI}(\lambda = 8)$$

$$P[X \le 3 \mid X \le 5] = \frac{f(0) + f(1) + f(2) + f(3)}{f(0) + f(1) + f(2) + f(3) + f(4) + f(5)}$$

$$= \frac{8^0/0! + 8^1/1! + 8^2/2! + 8^3/3!}{8^0/0! + 8^1/1! + 8^2/2! + 8^3/3! + 8^4/4! + 8^5/5!}$$

$$= \boxed{0.2216}$$

3. The number earthquakes in a certain region follows a Poisson distribution with a mean of 6 earthquakes per year. The probability that a given earthquake in the region has a magnitude greater than 3.0 is equal to 0.25, and is independent of the magnitude of other earthquakes. Find the probability that there are no earthquakes with a magnitude greater than 3.0 in a given year.

$\boxed{\text{A)}}$ 0.2231 B) 0.1830 C) 0.1964 D) 0.2097 E) 0.2365

$$P[\text{no EQ w/ mag} > 3.0] = \sum_{n=0}^{\infty} P[n \text{ EQ, all w/ mag} \le 3.0]$$

$$= \sum_{n=0}^{\infty} \frac{e^{-6}6^n}{n!}(0.75)^n = \sum_{n=0}^{\infty} \frac{e^{-6}(4.5)^n}{n!} = \frac{e^{-6}}{e^{-4.5}} \sum_{n=0}^{\infty} \frac{e^{-4.5}(4.5)^n}{n!}$$

Note that if $Z \sim \text{POI}(\lambda = 4.5)$, then $f(z) = \dfrac{e^{-z} z^n}{n!}$.

Thus $\displaystyle\sum_{n=0}^{\infty} \frac{e^{-4.5}(4.5)^n}{n!} = 1$.

$$P[\text{no EQ w/ mag} > 3.0] = \frac{e^{-6}}{e^{-4.5}} = \boxed{0.2231}$$

4. The number of hospital stays required for a patient with a certain condition follows a Poisson distribution with a mean of 2.9 stays per year. One such patient takes out an insurance policy to help her pay for potential hospital bills. The policy does not pay anything for the first two hospital stays in a given year, but pays 400 for each stay beyond the second. Find the expected amount paid by this insurance policy.

$\boxed{\text{A)}}$ 467.85 B) 425.74 C) 439.77 D) 453.81 E) 481.88

X = # of stays $X \sim POI(\lambda = 2.9)$

P = Amount paid by insurance. $P = \begin{cases} 0 & \text{if } X \le 2 \\ 400(X-2) & \text{if } X \ge 3 \end{cases}$

$E[P] = 400 f(3) + 800 f(4) + 1200 f(5) + \cdots$

$E[400(X-2)] = -800 f(0) - 400 f(1) + 0 \cdot f(0) + 400 f(3) + 800 f(4) + \cdots$

$= -800 f(0) - 400 f(1) + E[P]$

$E[P] = 800 f(0) + 400 f(1) + 400 E[X] - 800$

$= 800 \frac{e^{-2.9}\, 2.9^0}{0!} + 400 \frac{e^{-2.9}\, 2.9^1}{1!} + 400(2.9) - 800 = \boxed{467.85}$

5. Ivan works on an assembly line manufacturing widgets. Let X be the number of defective widgets that he produces in a given month. Assume X follows a Poisson distribution with a mean of 2.2. Ivan receives a monthly bonus of $25(X-4)$ if X is less than 4. If X is greater than 4, then Ivan receives no bonus for that month. Calculate the standard deviation of Ivan's monthly bonus.

$\boxed{\text{A)}}$ 31.60 B) 27.81 C) 29.07 D) 30.34 E) 32.86

X = # of defective widgets

B = Bonus amount

X	0	1	2	3	≥ 4
$f(x)$	0.1108	0.2438	0.2681	0.1966	0.1807
B	100	75	50	25	0

using 1-var stats: $\sigma = \boxed{31.60}$