HW 3.9(a) Key

1. Let X be a discrete random variable with moment generating function given by:
$M_X(t) = 0.2e^{-2t} + 0.18e^{-t} + 0.17 + 0.16e^{t} + 0.15e^{2t} + 0.14e^{3t}$
Find $P[X < 2 \mid X \geq 0]$.
$\boxed{\text{A)}}$ 0.5323 B) 0.5589 C) 0.5855 D) 0.6121 E) 0.6387

$$P[X<2 \mid X \geq 0] = \frac{P[0 \leq X \leq 1]}{P[X \geq 0]} = \frac{0.17+0.16}{0.17+0.16+0.15+0.14}$$

$$= \boxed{0.5323}$$

2. Let X be a discrete random variable with moment generating function given by:
$M_X(t) = 0.21 + 0.16e^{t} + 0.33e^{4t} + k \cdot e^{c \cdot t}$.
Given that $E[X] = 2.98$, find $Var[X]$.
$\boxed{\text{A)}}$ 4.0596 B) 4.2626 C) 4.4656 D) 4.6685 E) 4.8715

$$k = 1 - 0.21 - 0.16 - 0.33 = 0.3$$

$$E[x] = 0.21(0) + 0.16(1) + 0.33(4) + 0.3c = 2.98 \rightarrow c = 5$$

$$E[x^2] = 0.21(0)^2 + 0.16(1)^2 + 0.33(4)^2 + 0.3(5)^2 = 12.94$$

$$Var[x] = 12.94 - (2.98)^2 = \boxed{4.0596}$$

3. Let X be a discrete random variable with moment generating function given by: $M_X(t) = \dfrac{\ln(1-0.28e^{t})}{\ln(0.72)}$.
Find $Var[X]$.
$\boxed{\text{A)}}$ 0.2428 B) 0.2549 C) 0.2670 D) 0.2792 E) 0.2913

$$M_X'(t) = \frac{1}{\ln(0.72)} \frac{1}{1-0.28e^t}(-0.28e^t)$$

$$= \frac{1}{\ln(0.72)} \frac{-0.28e^t}{1-0.28e^t}$$

$$M_X''(t) = \frac{1}{\ln(0.72)} \frac{(1-0.28e^t)(-0.28e^t) - (-0.28e^t)(-0.28e^t)}{(1-0.28e^t)^2}$$

$$E[x] = M_X'(0) = \frac{1}{\ln(0.72)} \frac{-0.28}{0.72} = 1.1838$$

$$E[x^2] = M_X''(0) = \frac{1}{\ln(0.72)} \frac{0.72(-0.28) - (0.28)^2}{(0.72)^2} = 1.6442 \qquad Var[x] = \boxed{0.2428}$$

4. Let X be a discrete random variable with moment generating function given by: $M_X(t) = e^{-21+13e^t+8e^{-t}}$.

Find $Var[X]$.

$\boxed{\text{A)}}$ 21 B) 18 C) 19 D) 20 E) 22

$M_x'(t) = (13e^t - 8e^{-t}) M_x(t)$

$M_x''(t) = (13e^t + 8e^{-t}) M_x(t) + (13e^t - 8e^{-t}) M_x'(t)$

$M_x(0) = 1$

$E[x] = M_x'(0) = (13-8)(1) = 5$

$E[x^2] = M_x''(0) = (13+8)(1) + (13-8)(5) = 46$

$Var[x] = 46 - 25 = \boxed{21}$

5. Let X be a discrete random variable with moment generating function given by: $M_X(t) = \dfrac{0.65}{e^{-4t} - 0.35}$.

Find $Var[X]$.

$\boxed{\text{A)}}$ 13.2544 B) 12.5917 C) 13.9172 D) 14.5799 E) 15.2426

$M_x(t) = 0.65\left[e^{-4t} - 0.35\right]^{-1}$

$M_x'(t) = -0.65\left[e^{-4t} - 0.35\right]^{-2}(-4e^{-4t})$

$= 2.6e^{-4t}\left[e^{-4t} - 0.35\right]^{-2}$

$M_x''(t) = -10.4e^{-4t}\left[e^{-4t} - 0.35\right]^{-2} + 2.6(-2)(-4)e^{-4t}\left[e^{-4t} - 0.35\right]^{-3}$

$E[x] = M_x'(0) = 2.6(0.65)^{-2} = 6.1538$

$E[x^2] = M_x''(0) = -10.4(0.65)^{-2} + 20.8(0.65)^{-3} = 51.1243$

$Var[x] = 51.1243 - (6.1538)^2$

$= \boxed{13.2544}$