

HW 3.9(b) Key

1. Let X be a discrete random variable with moment generating function given by: $M_X(t) = (0.28 + 0.72e^t)^{23}$.

Find $\text{Var}[3X+12]$.

- A) 41.731 B) 33.385 C) 35.472 D) 37.558 E) 39.645

$$X \sim \text{BIN}(p=0.72, n=23)$$

$$\text{Var}[3X+12] = 9 \text{Var}[X] = 9(23)(0.72)(0.28) = \boxed{41.731}$$

2. Let X be a discrete random variable with moment generating function given by: $M_X(t) = \frac{0.34e^t}{1-0.66e^t}$.

Find $E[9X^2]$.

- A) 129.2388 B) 122.7768 C) 135.7007 D) 142.1626 E) 148.6246

$$X \sim \text{GEO}(p=0.34) \quad E[X] = 2.9412 \quad \text{Var}[X] = 5.7093$$

$$E[X^2] = 14.3599$$

$$E[9X^2] = \boxed{129.2388}$$

3. Let X be a discrete random variable with moment generating function given by: $M_X(t) = 0.2e^{3.8t} + 0.8e^{5.2t}$.

Find $\text{Var}[X^2]$.

- A) 25.402 B) 20.321 C) 21.591 D) 22.861 E) 24.132

$$M_X'(t) = 0.2(3.8)e^{3.8t} + 0.8(5.2)e^{5.2t}$$

$$M_X''(t) = 0.2(3.8)^2 e^{3.8t} + 0.8(5.2)^2 e^{5.2t}$$

$$M_X'''(t) = 0.2(3.8)^3 e^{3.8t} + 0.8(5.2)^3 e^{5.2t}$$

$$M_X^{(4)}(t) = 0.2(3.8)^4 e^{3.8t} + 0.8(5.2)^4 e^{5.2t}$$

$$E[X^2] = M_X''(0) = 24.52$$

$$E[X^4] = M_X^{(4)}(0) = 626.632$$

$$\text{Var}[X^2] = E[X^4] - (E[X^2])^2 = \boxed{25.4016}$$

4. Let X be a discrete random variable following a binomial distribution with $n = 5$ and $p = 0.65$. Let

$$Y = 25(3.5)^X. \text{ Find } E[Y].$$

- A) 3115.92 B) 3271.72 C) 3427.51 D) 3583.31 E) 3739.11

$$M_X(t) = (0.35 + 0.65e^t)^5$$

$$\begin{aligned} E[Y] &= E[25(3.5)^X] = 25 E[3.5^X] \\ &= 25 E[(e^{\ln 3.5})^X] = 25 E[e^{(\ln 3.5)X}] \\ &= 25 M_X(\ln 3.5) = 25(0.35 + 0.65(3.5))^5 \\ &= \boxed{3115.92} \end{aligned}$$

5. Let X be a discrete random variable following a geometric distribution with $p = 0.7$. Let $Y = 4X + 16$. Find

$$M_Y(t).$$

$$A) \quad M_Y(t) = \frac{0.7e^{20t}}{1 - 0.3e^{4t}}$$

$$M_X(t) = \frac{0.7e^t}{1 - 0.3e^t}$$

$$\begin{aligned} E[e^{tY}] &= E[e^{4Xt + 16t}] = e^{16t} E[e^{(4t)X}] \\ &= e^{16t} M_X(4t) \\ &= e^{16t} \frac{0.7e^{4t}}{1 - 0.3e^{4t}} \\ &= \boxed{\frac{0.7e^{20t}}{1 - 0.3e^{4t}}} \end{aligned}$$