HW 4.2(a) Key

1. Let X be a continuous random variable with probability density function given by $f(x) = \dfrac{Kx}{\left(9+x^2\right)^2}$ for $x \geq 0$, and 0 otherwise. Find $P[X > 5.2]$.

$u = 9 + x^2$
$du = 2x\,dx$

$\boxed{\text{A)}}$ 0.2497 B) 0.1898 C) 0.2048 D) 0.2198 E) 0.2347

$$\int f(x)\,dx = K\int \frac{x}{(9+x^2)^2}\,dx = \frac{K}{2}\int \frac{1}{u^2}\,du = \frac{K}{2}\left(-\frac{1}{u}\right) + C = \frac{-K}{2(9+x^2)} + C$$

$$\int_0^\infty f(x)\,dx = \left[-\frac{K}{2(9+x^2)}\right]_0^\infty = 0 - \left(-\frac{K}{18}\right) = 1 \Rightarrow K = 18$$

$$P[X > 5.2] = \int_{5.2}^\infty f(x)\,dx = \left[\frac{-9}{9+x^2}\right]_{5.2}^\infty = 0 - \left(-\frac{9}{9+(5.2)^2}\right) = \boxed{0.2497}$$

2. Let X be a continuous random variable with probability density function given by $f(x) = K x^{-1.3}$ if $x \geq 1$, and 0 otherwise. Find $P[X < 12.7 | X > 7.5]$.

0.1462

$\boxed{\text{A)}}$ ~~0.4958~~ B) 0.4660 C) 0.5255 D) 0.5553 E) 0.5850

$$\int_1^\infty Kx^{-1.3}\,dx = \left[\frac{K}{-0.3}x^{-0.3}\right]_1^\infty = 0 - \left(-\frac{K}{0.3}\right) = 1 \Rightarrow K = 0.3$$

$$F(x) = \int_1^x 0.3t^{-0.3}dt = \left[-t^{-0.3}\right]_1^x = \left[t^{-1.3}\right]_x^1 = 1 - x^{-0.3}$$

$$P[X < 12.7 | X > 7.5] = \frac{P[7.5 < X < 12.7]}{P[7.5]} = \frac{F(12.7) - F(7.5)}{1 - F(7.5)} = \boxed{0.1462}$$

3. Let X be a continuous random variable with probability density function given by $f(x) = \overset{\frac{1}{250}}{\cancel{K}}\, x^2 e^{-x/5}$ if $x \geq 0$, and 0 otherwise. Find $P[X < 17]$.

$\boxed{\text{A)}}$ 0.6603 B) 0.5810 C) 0.6206 D) 0.6999 E) 0.7395

		$e^{-x/5}$
+	$\frac{1}{250}x^2$	$-5e^{-x/5}$
−	$\frac{1}{125}x$	$25e^{-x/5}$
+	$\frac{1}{125}$	$-125e^{-x/5}$
−	0	*

$$P[X < 17] = \int_0^{17} f(x)\,dx$$

$$= -e^{-x/5}\left[0.02x^2 + 0.2x + 1\right]_0^{17}$$

$$= e^{-x/5}\left[0.02x^2 + 0.2x + 1\right]_{17}^0$$

$$= 1 - e^{-3.4}(10.18) = \boxed{0.6603}$$

4.
Let X be a continuous random variable with cumulative distribution function given by $F(x) = \dfrac{K}{4+e^{-x}}$ for $-\infty < x < \infty$. Find $P[X > -2.5 | X < -0.9]$.

A) 0.6008 B) 0.5648 C) 0.6369 D) 0.6729 E) 0.7090

$$F(\infty) = \lim_{x\to\infty} \frac{K}{4+e^{-x}} = \frac{K}{4} = 1 \Rightarrow K = 4$$

$$P[X > -2.5 | X < -0.9] = \frac{P[-2.5 < X < -0.9]}{P[X < -0.9]}$$

$$= \frac{F(-0.9) - F(-2.5)}{F(-0.9)} = \frac{\left(\frac{4}{4+e^{0.9}}\right) - \left(\frac{4}{4+e^{2.5}}\right)}{\left(\frac{4}{4+e^{0.9}}\right)}$$

$$= \boxed{0.6008}$$

5.
Let X be a continuous random variable with survival function given by $S(x) = e^{-x/2}\left(1+\dfrac{x}{2}\right)$ if $x \ge 0$, and 1 otherwise. Find $P[X > 9 | X > 4]$.

A) 0.1505 B) 0.1415 C) 0.1595 D) 0.1685 E) 0.1776

$$P[X > 9 | X > 4] = \frac{P[X > 9]}{P[X > 4]} = \frac{S(9)}{S(4)}$$

$$= \frac{0.06110}{0.40601} = \boxed{0.1505}$$