HW 4.3 Key

1\. Let X be a continuous random variable with probability density function given by $f(x)=\frac{6}{125}x(5-x)$ for $0<x<5$, and 0 otherwise. Find $Var[X]$.

$$f(x) = \frac{6x}{25} - \frac{6x^2}{125}$$

$\boxed{\text{A)}}$ 1.25 B) 1.15 C) 1.30 D) 1.40 E) 1.45

$$E[x] = \int_0^5 \left[\frac{6}{25}x^2 - \frac{6}{125}x^3\right]dx = \left[\frac{2}{25}x^3 - \frac{3}{250}x^4\right]_0^5 = 2.5$$

$$E[x^2] = \int_0^5 \left[\frac{6}{25}x^3 - \frac{6}{125}x^4\right]dx = \left[\frac{3}{50}x^4 - \frac{6}{625}x^5\right]_0^5 = 7.5$$

$$Var[x] = 7.5 - (2.5)^2 = \boxed{1.25}$$

2\. Let X be a continuous random variable with probability density function given by $f(x)=\frac{(1.6)^3}{2}x^2e^{-1.6x}$ if $x \geq 0$, and 0 otherwise. Find $E[X]$.

$\boxed{\text{A)}}$ 1.8750 B) 1.6500 C) 1.7625 D) 1.9875 E) 2.1000

		$(1.6)^3 e^{-1.6x}$
$+$	$\frac{1}{2}x^3$	$-(1.6)^2 e^{-1.6x}$
$-$	$\frac{3}{2}x^2$	$(1.6)\, e^{-1.6x}$
$+$	$3x$	$-e^{-1.6x}$
$-$	3	$\frac{1}{1.6}e^{-1.6x}$
$+$	0	$*$

$$E[x] = \int_0^\infty \frac{(1.6)^3}{2}x^3 e^{-1.6x}\,dx$$

$$= \left[\left(\frac{(1.6)^2}{2}x^3 + \frac{4.8}{2}x^2 + 3x + \frac{3}{1.6}\right)e^{-1.6x}\right]_\infty^0$$

$$= \frac{3}{1.6} - 0 = \boxed{1.875}$$

3. Let T denote the time in years until failure for a new electrical component in a machine. Assume that T follows a continuous distribution with pdf given by $f(t) = \frac{1}{64} t e^{-t/8}$ if $t \geq 0$, and 0 otherwise. The component will be replaced when it fails, or after 11 years, whichever occurs first. Let X denote the amount of time passed until the component is replaced. Find $E[X]$.

$\boxed{\text{A)}}$ 9.173 B) 8.595 C) 9.751 D) 10.329 E) 10.907

$$E[x] = \int_0^{11} t\left(\tfrac{1}{64} t e^{-t/8}\right) dt + \int_{11}^{\infty} 11\left(\tfrac{1}{64} t e^{-t/8}\right) dt$$

$$= \left[\left(\tfrac{1}{8}t^2 + 2t + 16\right) e^{-t/8}\right]_{11}^{0} + 11\left[\left(\tfrac{1}{8}t + 1\right) e^{-t/8}\right]_{\infty}^{11}$$

$$= \left[16 - 53.125\, e^{-1.375}\right] + 11\left[2.375\, e^{-1.375} - 0\right]$$

$$= \boxed{9.1733}$$

		$\frac{1}{64} e^{-t/8}$
+	t^2	$-\frac{1}{8} e^{-t/8}$
−	$2t$	$e^{-t/8}$
+	2	$-8 e^{-t/8}$
−	0	*

		$\frac{1}{64} e^{-t/8}$
+	t	$-\frac{1}{8} e^{-t/8}$
−	1	$e^{-t/8}$
+	0	*

4. Let X be a continuous random variable with probability density function given by $f(x) = 2x^{-3}$ if $x \geq 1$, and 0 otherwise. Let $Y = 4\sqrt{X}$. Find $Var[Y]$.

$\boxed{\text{A)}}$ 3.556 B) 2.702 C) 2.916 D) 3.129 E) 3.342

$$E[Y] = E[4\sqrt{x}] = \int_1^{\infty} 4x^{\frac{1}{2}}(2x^{-3})\, dx = 8\int_1^{\infty} x^{-2.5}\, dx$$

$$= \frac{8}{-1.5} x^{-1.5} \Big|_1^{\infty} = \frac{8}{1.5} = 5.3333$$

$$E[Y^2] = E[16x] = \int_1^{\infty} 16x(2x^{-3})\, dx = 32\int_1^{\infty} x^{-2}\, dx$$

$$= 32x^{-1}\Big|_1^{\infty} =$$

$$Var[Y] = 32 - (5.3333)^2 = \boxed{3.5556}$$

5. Let X be a continuous random variable with probability density function given by $f(x) = 0.9x^{-1.9}$ if $x \geq 1$, and 0 otherwise. Let $Y = \ln(X^6)$. Find $E[Y]$.

$\boxed{\text{A)}}$ 6.667 B) 5.867 C) 6.267 D) 7.067 E) 7.467

$$E[Y] = E[\ln(x^6)] = \int_1^\infty \ln(x^6)[0.9x^{-1.9}]\,dx = \int_1^\infty 6\ln x(0.9x^{-1.9})\,dx$$

$$= 5.4\int_1^\infty \ln x\,(x^{-1.9})\,dx$$

$$= 5.4\left[\frac{\ln x}{0.9}x^{-0.9} + \frac{1}{(0.9)^2}x^{-0.9}\right]_\infty^1$$

$$= 5.4\left[\frac{1}{(0.9)^2} - 0\right] = \boxed{6.667}$$

$u = \ln x$ $\quad v = \frac{1}{-0.9}x^{-0.9}$

$du = \frac{1}{x}$ $\quad dv = x^{-1.9}dx$

$$\int \ln x(x^{-1.9})\,dx = -\frac{\ln x}{0.9}x^{-0.9} + \frac{1}{0.9}\int x^{-1.9}\,dx$$

$$= -\frac{\ln x}{0.9}x^{-0.9} - \frac{1}{(0.9)^2}x^{-0.9} + C$$